RADIOSS THEORY MANUAL
Version 2017– January 2017
ALE

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1

ARBITRARY LAGRANGIAN EULERIAN FORMULATION

1.0 ALE FORMULATION

ALE or Arbitrary Lagrangian Eulerian formulation is used to model the interaction between fluids and solids; in particular, the fluid loading on structures. It can also be used to model fluid-like behavior, as seen in plastic deformation of materials.

ALE derives its name from a combination of two different finite element modelling techniques.

- Lagrangian Formulation - where the observer follows material points.

- Eulerian Formulation - where the observer looks at fixed points in space.

- Arbitrary Lagrangian Eulerian Formulation - where the observer follows moving points in space.

1.1 Referential Domain

At any location in space x and time t, there is one material point, identified by its space coordinates x at time t=0, and one grid point identified by its coordinates ξ at time t=0. Figure 1.1.1 provides a pictorial representation and defines the velocities in each formulation.

Figure 1.1.1 ALE Formulation
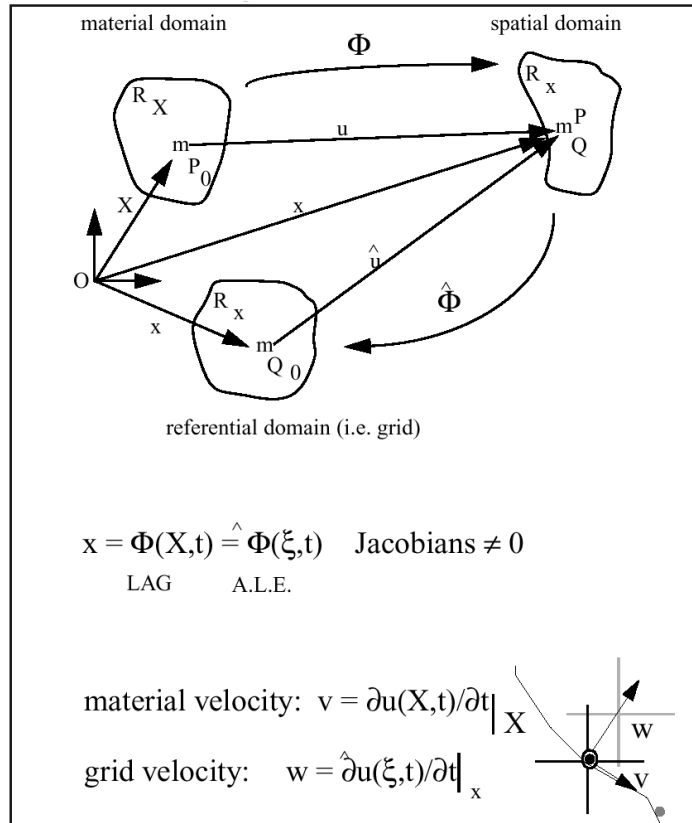
The derivative of any physical quantity can be computed either following the material point or following the grid point. They can then be related to each other.

Given that F is a function f of space and time representing a physical property:

The spatial domain is given by f(x,t).

The material domain is given by f*(X,t).

The mixed domain is given by f**(ξ, t).

Therefore:

$$\frac{\partial f^*}{\partial t}\Big|_X = \frac{\partial f}{\partial t}\Big|_X + \frac{\partial f}{\partial x_j} \times \frac{\partial x_j}{\partial t}\Big|_X = \frac{\partial f}{\partial t}\Big|_X + V_j(x_k,t)\frac{\partial f(x,t)}{\partial x_j}\Big|_t \qquad \text{EQ. 1.1.0.1}$$

Also:

$$\frac{\partial f^*}{\partial t}\Big|_X = \frac{\partial f^{**}}{\partial t}\Big|_\xi + (V_j - W_j)\frac{\partial f(x,t)}{\partial x_j}\Big|_t \qquad \text{EQ. 1.1.0.2}$$

This relates to acceleration by:

$$\vec{\gamma} = \frac{d\vec{v}}{dt}\Big|_X = \frac{\partial}{\partial t}\vec{v}\Big|_\xi + (v_j - w_j)\frac{\partial}{\partial x}\vec{v}\Big|_t \qquad \text{EQ. 1.1.0.3}$$

where V = material velocity

w = grid velocity

1.2 Conservation of Momentum

Conservation of momentum, expressed in terms of a finite element formulation, is given by:

$$\int_V \Phi_I \left(\rho\frac{\partial v_i}{\partial t} - \frac{\partial \sigma_{ij}}{\partial x_j} - \rho b_i \right) dV = 0 \qquad \text{EQ. 1.2.0.4}$$

where Φ_I = the weight functions

ρ = Material density

V = Velocity

σ_{ij} = Stress Matrix

b_i = Body acceleration vector

V = Volume

This can be rewritten in a form similar to the explicit Lagrangian formulation with the addition of a new nodal force f_{trm}, accounting for transport of momentum:

$$M\frac{\partial}{\partial t}v = \{F^{ext}\} - \{F^{int}\} + \{F^{bod}\} + \{F^{hgr}\} + \{F^{trm}\} \qquad \text{EQ. 1.2.0.5}$$

where $\{F^{trm}\} = \sum f^{trm}$

The transport of momentum force is calculated by:

$$f_{iI}^{trm} = (1+\eta_I)\int_V \rho\Phi_I(w_j - v_j)\frac{\partial v_i}{\partial x_j}dV \qquad \text{EQ. 1.2.0.6}$$

where i,j = direction index

I = connectivity

η_I = Upwind factor

1.2.1 Momentum transport force

Momentum transport forces are computed using the relation:

$$F_{il}^{trm} = \left(1 + \eta_I\right)\rho\Phi_I\left(w_j - v_j\right)\frac{\partial v_i}{\partial x_j}V \qquad \text{EQ. 1.2.1.1}$$

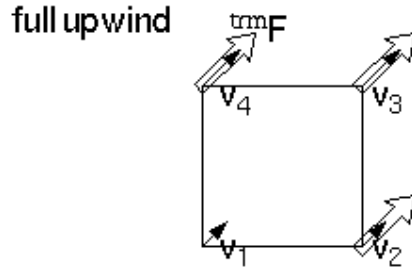The upwinding technique is introduced to add numerical diffusion to the scheme, which otherwise is generally under diffuse and thus unstable.

$$\eta_I = \eta\, sign\left[\frac{\partial \Phi_I}{\partial x_j}\left(v_j - w_j\right)\right] \qquad \text{EQ. 1.2.1.2}$$

$0 \le \eta \le 1$ Upwind coefficient, given in input.

Full upwind $\eta = 1$ (default value) is generally used.

Development of less diffusive flux calculation is currently under investigation.

1.3 Conservation of Mass

The finite element formulation of the Lagrangian form of the mass conservation equation is given by:

$$\frac{d\rho}{dt}\Big|_X = -\left(\rho/V\right)\frac{dV}{dt}\Big|_X \qquad \text{EQ. 1.3.0.2}$$

When transformed into the ALE formulation it gives:

$$\frac{\partial \rho}{\partial t}\Big|_\xi - \left(\left(w_i - v_i\right)\cdot\frac{\partial \rho}{\partial x_i}\Big|_t\right) + \rho\frac{\partial v_K}{\partial x_K}\Big|_t = 0 \qquad \text{EQ. 1.3.0.3}$$

Applying a Galerkin variation form for the solution of equation 1.3.0.3:

$$\int_V \psi\left(\frac{\partial \rho}{\partial t}\Big|_\xi - \left(\left(w_i - v_i\right)\cdot\frac{\partial \rho}{\partial x_i}\Big|_t\right) + \rho\frac{\partial v_K}{\partial x_K}\Big|_t\right) = 0 \qquad \text{EQ. 1.3.0.4}$$

where ψ = Weighting function

Using a finite volume formulation:

where $\psi = 1$

ρ = constant density over control volume V

Therefore:

$$\int_V \frac{\partial \rho}{\partial t} dV + \int_V \rho \frac{\partial v_K}{\partial x_K} dV = 0 \qquad \text{EQ. 1.3.0.5}$$

Using the divergence theorem leads to:

$$\int_V \frac{\partial \rho}{\partial t} dV + \int_S \rho \left(v_j \cdot n_j\right) dS = 0 \qquad \text{EQ. 1.3.0.6}$$
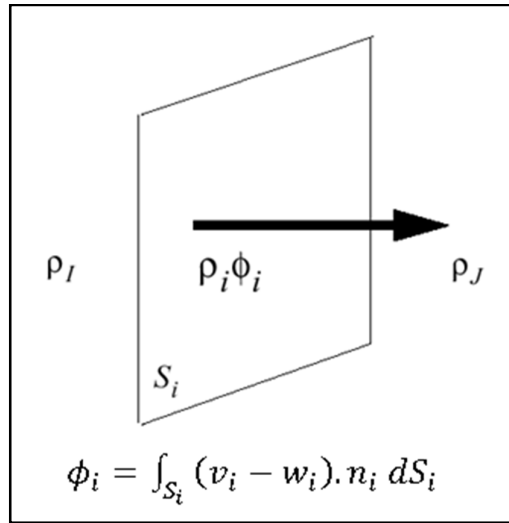
Further expansion gives:

$$\frac{d}{dt} \int_V \rho dV = \int_S \underbrace{\rho \left(w_j - v_j\right) n_j dS}_{mass\ flux} \qquad \text{EQ. 1.3.0.7}$$

This formula is still valid if density ρ is not assumed uniform over volume V.

The mass flux across a surface is shown in Figure 1.3.1.

Figure 1.3.1 Mass Flux

$$\phi_i = \int_{S_i} (v_i - w_i). n_i \, dS_i$$

The density, ρ_i, is given computed:

$$\rho_i = \tfrac{1}{2}\rho_I\{1 + \eta\ sign(\phi_i)\} + \tfrac{1}{2}\rho_J\{1 - \eta\ sign(\phi_i)\} \qquad \text{EQ. 1.3.0.8}$$

where $0 \leq \eta \leq 1$ is the upwind coefficient given on the input card.

If $\eta = 0$, there is no upwind. Therefore: $\rho_i = \dfrac{\rho_I + \rho_J}{2}$

If $\eta = 1$, there is full upwind.

The smaller the upwind factor, the faster the solution; however, the solution is more stable with a large upwind factor.

For a free surface: $\rho_J = \rho_I$

1.4 Conservation of Internal Energy

Conservation of internal energy is used to model temperature dependent material behavior. It also allows an energy balance evaluation. However, internal energy is only calculated if it is turned on, to reduce computation time in problems not involving heat transfer.

The conservation of energy is given by:

$$\frac{\partial \rho e}{\partial t} - \left((w_i - v_i) \cdot \frac{\partial \rho e}{\partial x_i} \right) + (\rho e + P) \frac{\partial v_K}{\partial x_K} = 0 \qquad\qquad \text{EQ. 1.4.0.9}$$

Where, e = Internal energy in Joules (Nm)

$\qquad$ P = Fluid pressure

Applying a Galerkin variation form for the solution gives:

$$\int_V \psi \left(\frac{\partial \rho e}{\partial t} - \left((w_i - v_i) \cdot \frac{\partial \rho e}{\partial x_i} \right) + (\rho e + P) \frac{\partial v_K}{\partial x_K} \right) = 0 \qquad\qquad \text{EQ. 1.4.0.10}$$

Making the following assumptions:

$\psi = 1$

ρe = constant over control volume V

Equation 1.4.0.10 reduces to:

$$\int_V \frac{\partial \rho e}{\partial t} dV + \int_V (\rho e + P) \frac{\partial v_K}{\partial x_K} dV = 0 \qquad\qquad \text{EQ. 1.4.0.11}$$

Applying the divergence theorem gives:

$$\int_V \frac{\partial \rho e}{\partial t} dV + \int_S \left(\rho e (v_j \cdot n_j) dS \right) + \int_V P \frac{\partial v_K}{\partial x_K} dV = 0 \qquad\qquad \text{EQ. 1.4.0.12}$$

Hence:

$$\frac{d}{dt} \int_V \rho e \, dV = \int_S \rho e (w_j - v_j) n_j dS - \int_V P \frac{\partial v_K}{\partial x_K} dV \qquad\qquad \text{EQ. 1.4.0.13}$$

This formula is still valid if ρe is not assumed uniform over volume V.

1.5 Rezoned Quantities

The deviatoric stress tensor and the equivalent plastic strain must be rezoned and recalculated after every time step due to the ability of one element to contain a different amount of material.

1.6 ALE Materials

The following materials may be used with the ALE formulation.

| ALE Materials	
Law Number	**Description**
2	Elasto-plastic /MAT/PLAS_JOHN
3	Elasto-plastic-Hydrodynamic /MAT/HYDPLA
4	Johnson Cook /MAT/HYD_JCOOK
6	Hydrodynamic Viscous /MAT/HYD_VISC
10 and 21	Rock Concrete Foam /MAT/LAW10 or /MAT/DPRAG
22 and 23	Elasto-plastic with Damage /MAT/DAMA or /MAT/LAW23
20	Bimaterial /MAT/BIMAT
37	Hydrodynamic - Bi-phase liquid gas /MAT/BIPHAS
11	Boundary - Stagnation conditions in flow calculations /MAT/BOUND
16	Gray model - Multiphase Gray E.O.S + Johnson's shear law /MAT/GRAY
18	Thermal conductivity, purely thermal material /MAT/THERM

For the rest, refer to the next version of the theory manual.

1.7 Numerical Integration

The numerical integration techniques used are the same as those used for any other analysis type.

The flow chart of calculations can be seen in Figure 1.7.1.

Figure 1.7.1 Flow Chart[1]
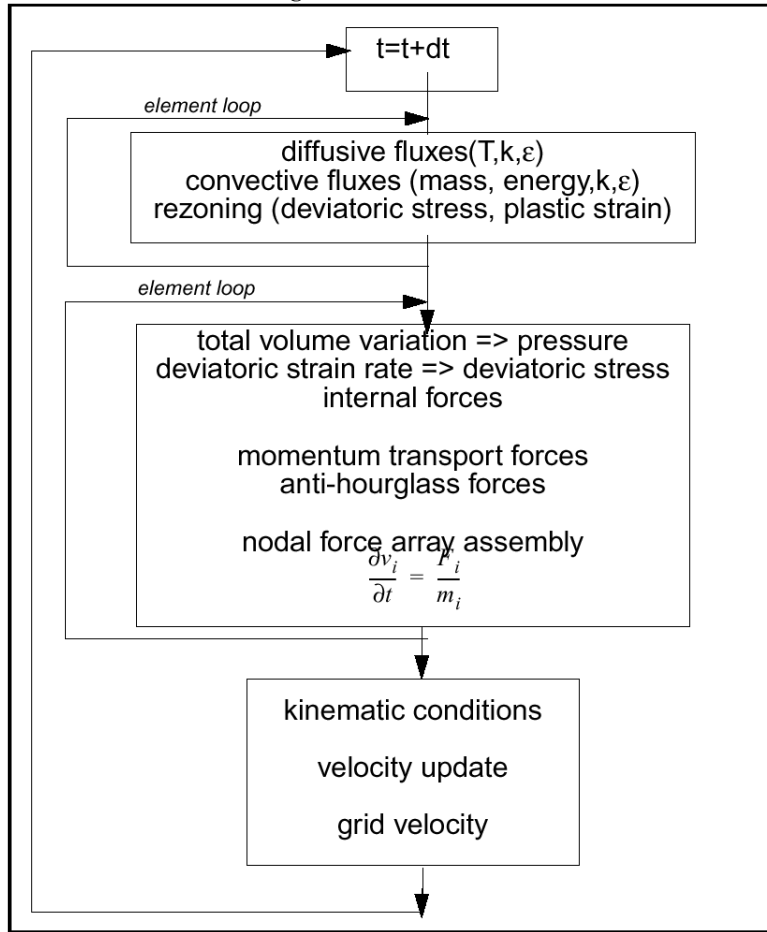
1. Simplified Flow Chart

1.8 Improved integration method

This method can only be used with the CFD version of RADIOSS, and only available in Eulerian formulation. An eight Gauss point integration scheme is used to determine the shape functions. The shape functions are condensed to one point. This gives an eight point integration scheme with constant stress.

1.9 Momentum Transport Force

This scheme is only used with the ALE formulation (Arbitrary Lagrangian Eulerian) and in the CFD version of RADIOSS. The force is calculated using the relation:

$$^{trm}F_i^I = \left(1 + \eta_I\right)\rho\Phi_I\left(w_j - v_j\right)\frac{\partial v_i}{\partial X_j}V \; .$$

EQ. 1.9.0.14

Where, w = grid velocity

ν = material velocity

V = element volume

η = upwind coefficient (user defined, default = 1 for full upwind)

When a Lagrangian formulation is used, the values of w_j and v_j are equal. Thus, EQ. 1.9.0.14 is equal to zero.

1.9.1 UPWINDING TECHNIQUE

An upwinding technique is introduced to add numerical diffusion to the scheme; otherwise it is generally under diffusive and thus unstable. The upwind coefficient used in EQ. 1.9.0.14 is calculated by:

$$\eta_I = \eta sign\left(\frac{\partial \Phi_I}{\partial X_j}\left(v_j - w_j\right)\right) \qquad \text{EQ. 1.9.1.1}$$

Development of a less diffusive flux calculation is currently under investigation.

$$F_i^I = \sigma_{ij}\int_V \frac{\partial \Phi_I}{\partial X_j} dV \qquad \text{EQ. 1.9.1.2}$$

This option is activated with the flag INTEG (only in the CFD version).

1.10 Stability

The Courant condition (neglecting viscosity effects) is used to determine the stability of an ALE process. The maximum time step is calculated by:

$$\Delta t \leq k \frac{\Delta l}{c + v - w} \qquad \text{EQ. 1.10.0.3}$$

Where, k = coefficient

Δl = Smallest characteristic length of an element

c = Material speed of sound

v = Material velocity

w = Grid velocity

The speed of sound is determined by:

$$c = \sqrt{\frac{1}{\rho}\frac{\partial \rho}{\partial \rho} + \frac{4}{3}\frac{\mu}{\rho}} \qquad \text{EQ. 1.10.0.4}$$

Where, ρ = Density

μ = Dynamic viscosity

p = Pressure

The relative velocity between the material and grid motion (v-w) is computed by:

$$v - w = \sqrt{\frac{1}{N}\sum_{i=1}^{3}\sum_{I=1}^{N}\left(v_i^I - w_i^I\right)^2} \qquad \text{EQ. 1.10.0.5}$$

Where, N = Number of nodes of the considered element (usually N=8)

1.11 ALE Kinematic Conditions

1.11.1 Boundary Conditions

Boundaries with Lagrangian materials are declared automatically Lagrangian.
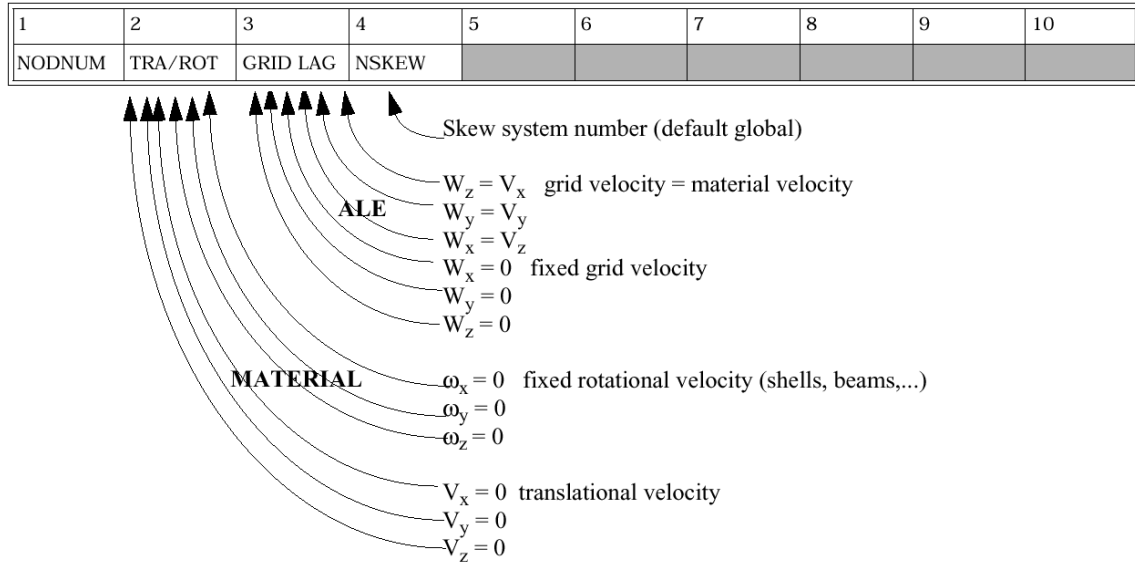
Nodes can be declared Lagrangian.

Constraints can be applied separately or simultaneously on:

- Material velocity
- Grid velocity

These constraints can be applied in one or several directions of a skew reference frame.

When the flag is set to 1, boundary condition is activated with global reference frame or skew reference frame.

i = d.o.f with respect to global reference frame or skew reference frame

VELOCITY: $V_i = 0$

ACCELERATION: $\prod i = 0$

The boundary conditions can be changed during Engine runs with /BCS or /BCSR Engine options.

1.11.2 ALE Links

An ALE link is identical to a rigid link. The slave node sets' grid velocity can be controlled by two master nodes, M1 and M2.

There are three options to choose from:

Option 0:

Velocity is linearly interpolated with respect to order of input.

$$W_{NI} = W_{M1} + (W_{M2} - W_{M12})\frac{I}{N+1}$$ EQ. 1.11.2.1

Option 1:

Velocity is set to maximum absolute velocity of master nodes.

$$W_{NI} = W_{M1} \qquad \text{if} \qquad |W_{M1}| > |W_{M2}|$$ EQ. 1.11.2.2

Option 2:

Velocity is set to minimum absolute velocity of master nodes.

$$W_{NI} = W_{M1} \qquad \text{if} \qquad \left|W_{M1}\right| < \left|W_{M2}\right|$$ EQ. 1.11.2.3

The input data is specified at each restart run.

1.12 Automatic Grid Computation

There are three different grid velocity formulations that can be used in an ALE simulation. New keywords define the type of method used. The different formulations are:

- 0 - J. Donea Grid Formulation: use keyword /DONEA

  (NWALE =0 for version < 4.1)

- 1 - Average Displacement Formulation: use keyword /DISP

  (NWALE =1 for version < 4.1)

- 2 - Nonlinear Spring Formulation: use keyword /SPRING

  (NWALE =2 for version < 4.1)

1.12.1 /DONEA - J. Donea Grid Formulation

This formulation [8], [72] computes grid velocity using:

$$W_I\left(t + \Delta t / 2\right) = \frac{1}{N}\sum_J W_J\left(t - \Delta t / 2\right) + \frac{1}{N^2}\frac{\alpha}{\Delta t}\sum_J L_{IJ}(t)\sum_J \frac{u_J(t) - u_I(t)}{L_{IJ}(t)}$$ EQ. 1.12.1.1

where, $1 - \gamma \leq \dfrac{w}{v} \leq 1 + \gamma$

N = Number of nodes connected to node I

L_{IJ} = Distance between node I and node J

α, γ = adimensional factors given in input

Therefore, the grid displacement is given by:

$$u\left(t + \Delta t\right) = u\left(t\right) + w\left(t + \Delta t / 2\right)\Delta t$$ EQ. 1.12.1.2

## 1.12.2 /DISP - Average Displacement Formulation

The average displacement formulation calculates average velocity to determine average displacement.

$$u\left(t + \Delta t\right) = \frac{1}{N}\sum_J w_j(t)$$ EQ. 1.12.2.1

1.12.3 /SPRING - Nonlinear Spring Formulation

Each grid node is connected to neighboring grid nodes through a non-linear viscous spring, similar to that shown in Figure 1.12.1.

Figure **1.12.1** Spring Force Graph
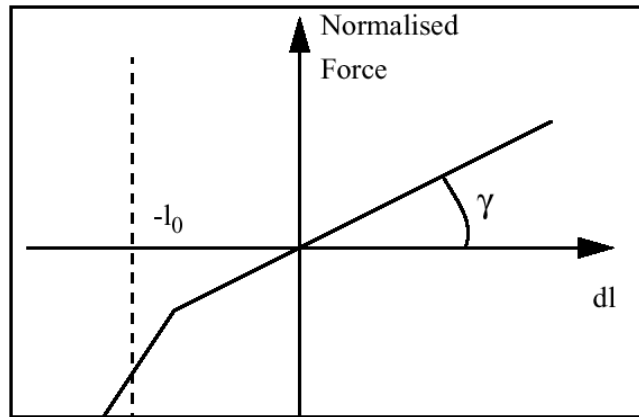
The input parameters required are:

ΔT_0 = typical time step (Must be greater than the time step of the current run.)

$0 < \gamma < 1$ = Nonlinearity factor

η = Damping coefficient

v= Shear factor (stiffness ratio between diagonal springs and springs along connectivities)

This formulation is the best of the three, but it is the most computationally expensive.

1.13 Type 1 interface - Fluid-Structure Interaction

Type 1 interface is used to model fluid-structure interactions, as shown in Figure 1.13.1

Figure 1.13.1 Fluid-Structure Interaction
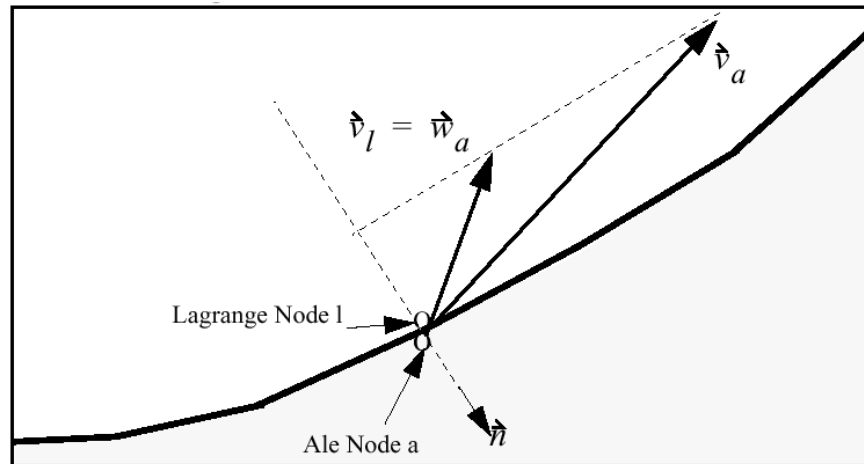
This interface allows Lagrangian elements (structure) to interact with ALE (Arbitrary Lagrangian Eulerian) elements, which model a viscous fluid. Full slip conditions are applied at the boundary between the two domains.

The acceleration of the Lagrange node is computed by:

$$\vec{\gamma}_l = \frac{\vec{F}_l + \vec{F}_a}{m_l + m_a}$$

EQ. 1.13.0.1

The acceleration of the ALE node is computed by:

$$\vec{\gamma}_a = \frac{\vec{F}_a}{m_a}$$

EQ. 1.13.0.2

The grid velocity of the ALE node is equal to the material velocity of the Lagrange node:

$$\vec{w}_a = \vec{v}_l$$

EQ. 1.13.0.3

The normal material velocities of Lagrange and ALE nodes are equal. Therefore:

$$\vec{v}_a \cdot \vec{n} = \vec{v}_l \cdot \vec{n}$$

EQ. 1.13.0.4

1.14 ALE Rigid Wall

An ALE rigid wall has similar properties to other types of rigid walls. There are two different types:

1. Shaped Charged: use keyword /DFS/WALL_SHAP

2. Penetration: use keyword /DFS/WALL_PEN

For further explanation of each rigid wall, contact Altair Development France.

Impacting nodes can either have a sliding contact or be tied to the rigid surface contact point. The wall can also be moving.

An example of an object impacting an ALE rigid wall can be seen in Figure 1.14.1.

Figure 1.14.1 ALE Rigid Wall Impact
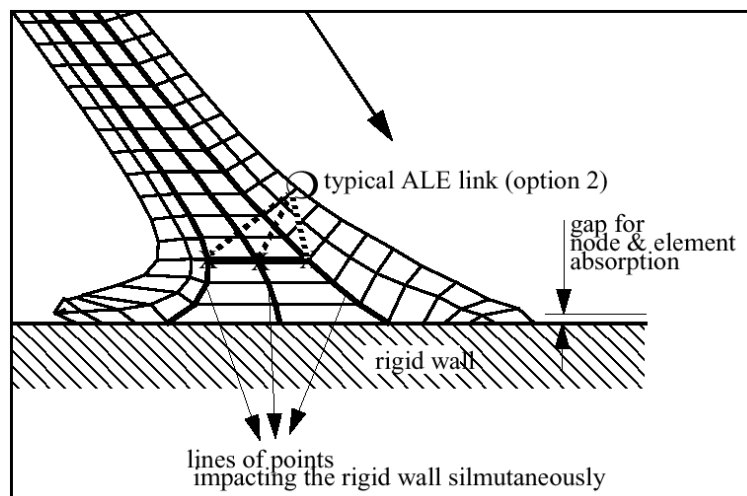
A gap is required for the wall, Figure 1.14.1. When a slave node distance to the rigid wall is within the gap:

- Nodes are forced onto the rigid wall and set as Lagrangian.
- Zero volume elements are emptied into neighboring elements and deleted.

In addition to the information explained above, an ALE rigid wall definition requires the number of nodes impacting simultaneously to be defined, along with the order of slave node impact.

1.15 Example

A typical application of the ALE method is using high velocity impacts. Below, a cylinder, moving at 227 m/s, impacts with a rigid wall. The material is copper, with a yield stress of 400 MPa. The initial diameter is 6.4 mm and initial length is 32.4 mm. The simulation was performed using two different methods: ALE and standard Lagrangian. The results can be seen in Figure 1.15.1.

Figure 1.15.1 Cylinder Impact Deformation
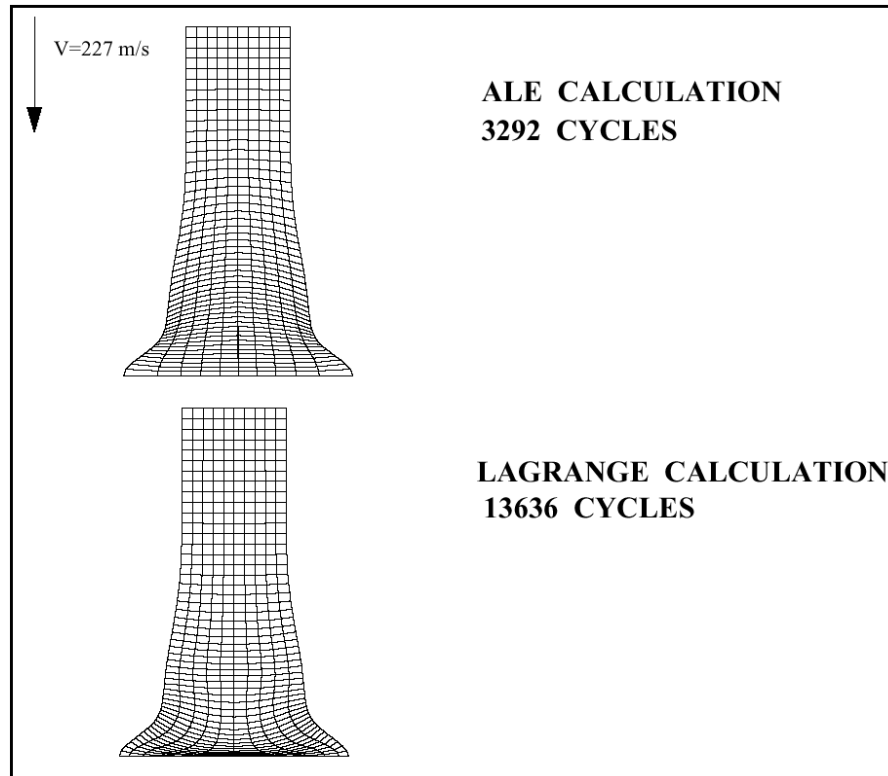
It can be seen that the cylinder mesh using ALE remains regular, unlike the Lagrange method, where large element deformation creates very small and skewed elements. This reduces the time step, leading to more time step cycles. However, each ALE cycle takes longer than a Lagrangian.