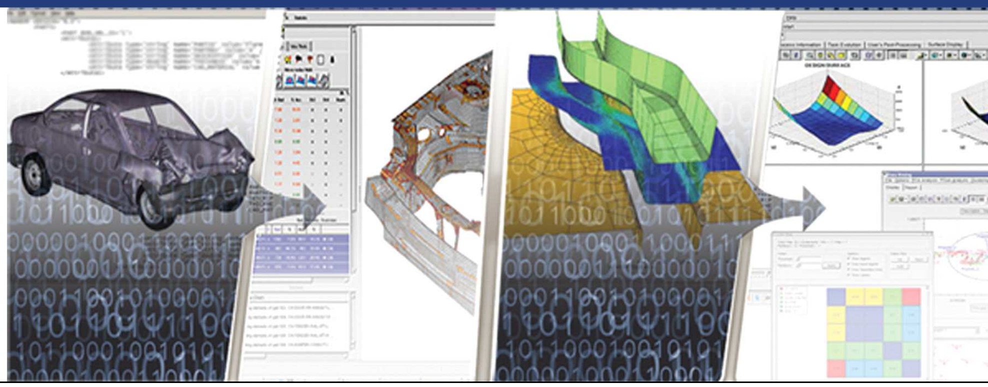

RADIOSS THEORY MANUAL

Version 2017 – January 2017

Large Displacement Finite Element Analysis

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14.0 APPENDICES

Appendix A
Conversion Tables & Constants

Length			
US & UK >> Metric system		**Metric system >> US & UK**	
1 Inch (in) - US	25.40005 mm	1 millimeter (mm)	0.03937 in (US)
1 Inch (in) - UK	25.39996 mm	1 millimeter (mm)	0.03937 in (UK)
1 Foot (ft) = (12.in) - US	0.3048006 m	1 meter (m)	3.28083 ft (US)
1 Foot (ft) = (12.in) - UK	0.3047995 m	1 meter (m)	3.28083 ft (UK)
1 Yard (yd) = (3.ft) - US	0.9144018 m	1 meter (m)	1.093611 yd (US)
1 Yard (yd) = (3.ft) - UK	0.9143984 m	1 meter (m)	1.093611 yd (UK)
1 Mile (mi) = (1760.yd) - US	1.609347 km	1 kilometer (km)	0.6213699 mi (US)
1 Mile (mi) = (1760.yd) - UK	1.609341 km	1 kilometer (km)	0.6213724 mi (UK)
1 Nautical mile (UK)	1.853181 km	1 kilometer (km)	0.5396127 n.mi (UK)

Surface			
US & UK >> Metric system		**Metric system >> US & UK**	
1 Acre - US	0.4046873 ha	1 hectare (ha)	2.471044 acre (US)
1 Acre - UK	0.4046842 ha	1 hectare (ha)	2.4711 acre (UK)
1 Square inch (sq in) - US	6.451626 cm^2	1 Square centimeter (cm$^{2)}$	0.1549997 sq. in (US)
1 Square inch (sq in) - UK	6.451578 cm^2	1 Square centimeter (cm$^{2)}$	0.1550 sq.in (UK)
1 Square foot (sq ft) = 144 sq in - US	0.09290341 m^2	1 Square meter (m$^{2)}$	10.76387 sq.ft (US)
1 Square foot (sq ft) = 144 sq in - UK	0.09290272 m^2	1 Square meter (m$^{2)}$	10.7639 sq.ft (UK)
1 Square yard (sq yd) = 9 sq.ft - US	0.8361307 m^2	1 Square meter (m$^{2)}$	1.195985 sq.yd (US)
1 Square yard (sq yd) = 9 sq.ft - UK	0.8361245 m^2	1 Square meter (m$^{2)}$	1.1960 sq.yd (UK)
1 Square mile (sq mi) = 640 acres - US	2.589998 km^2	1 Square kilometer (km$^{2)}$	0.3861006 sq.mi (US)
1 Square mile (sq mi) = 640 acres - UK	2.589979 km^2	1 Square kilometer (km$^{2)}$	0.3861 sq.mi (UK)

Volume			
US/UK >> Metric system		**Metric system >> US/UK**	
Volume			
1 Cubic inch (cu in) - US	16.3871 cm3	1 Cubic centimeter (cm$^{3)}$	0.06102509 cu in (US)
1 Cubic inch (cu in) - UK	16.38698 cm3	1 Cubic centimeter (cm$^{3)}$	0.0610241 cu in (UK)
1 Cubic foot (cu ft) - US	28.31702 dm3	1 Cubic decimeter (dm^3)	0.03531544 cu ft (US)
1 Cubic foot (cu ft - (UK)	28.31670 dm3	1 Cubic decimeter (dm^3)	0.0353148 cu ft (UK)
1 Cubic yard (cu yd) - US	0.7645594 m^3	1 Cubic meter (m$^{3)}$	1.307943 cu yd (US)
1 Cubic yard (cu yd) - UK	0.7645509 m^3	1 Cubic meter (m$^{3)}$	1.307957 cu yd (UK)

Measure of capacity			
1 fluid ounce (fl oz) - US	29.5735 cm3 (or ml)	1 Cubic decimeter (dm^3)	33.814 fl oz
1 fluid ounce (fl oz) - UK	28.4131 cm3 (or ml)	1 Cubic decimeter (dm^3)	35.195 fl oz
1 Bushel (US)	35.23829 dm3 (or liter)	1 Cubic decimeter (dm^3)	0.0283782 bu (US)
1 Bushel (UK)	36.36770 dm3 (or liter)	1 Cubic decimeter (dm^3)	0.02749692 bu (UK)
1 Gallon (US)	3.785329 dm3 (or liter)	1 Cubic decimeter (dm^3)	0.2641779 gal (US)
1 Gallon (UK)	4.545963 dm3 (or liter)	1 Cubic decimeter (dm^3)	0.2199754 gal (UK)
1 Liquid pint (US)	0.4731661 dm3 (or liter)	1 Cubic decimeter (dm^3)	2.113423 liq.pt (US)
1 Pint(pt) = 20 fl oz - UK	0.5682454 dm3 (or liter)	1 Cubic decimeter (dm^3)	1.759803 pt (UK)

Mass

Notes :

- *Attention should be paid to not confuse mass and weight.*
- *The mass (kg) is an intrinsic characteristic of the body and is measured in kilogram.*
- *The weight is a force which depends on terrestrial attraction and it is the equivalent of the mass of a body by the acceleration of gravity (9.80665 at the sea level) and is measured in Newton [N].*
- *The specific mass is the quotient of the mass of a body by its volume.*
- *The unit of mass of S.I. = kilogram.*

US/UK >> Metric system		**Metric system >> US/UK**	
1Grain (gr) - US	64.79892 mg	1 milligram (mg)	0.01543236 gr (US)
1Grain (gr) - UK	64.79892 mg	1 milligram (mg)	0.01543236 gr (UK)
1Ounce (oz) - US	28.34953 g	1 gram (g)	0.03527396 oz av. (US)
1Ounce (oz) - UK	28.34953 g	1 gram (g)	0.03527396 oz av. (UK)
1Pound (Ib) = 16 oz - US	0.4535924 kg	1 kilogram (kg)	2.204622 lb av. (US)
1Pound (Ib) = 16 oz - UK	0.4535924 kg	1 kilogram (kg)	2.204622 lb av. (UK)
1Short hundredweight(sh cwt)= 100 Ib - US	45.35924 kg	1kilogram (kg)	0.02204622 sh.cwt (US)
1Cental (UK)	45.35924 kg	1 kilogram (kg)	0.02204622 ctl (UK)
1Long ton (l tn) = 2240 Ib - US	1.016047 t	1 ton	0.9842064 l.tn (US)
1Ton (UK)	1.016047 t	1 ton	0.9842064 tn (UK)

Speed (the unit speed of the S.I = meter per second

Symbol	SI equivalent	Designation of unit measure
1 Bz	= 1 m/s	- benz
1 cm/min	= 1.66667 x 10-4 m/s	- centimeter per minute
1 cm/s	= 0.01 m/s	- centimeter per second
1 ft/h, fph	= 8.46667 x 10-5 m/s	- foot per hour
1 ft/min, fpm	= 0.00508 m/s	- foot per minute
1 ft/s, fps	= 0.3048 m/s	- foot per second
1 furlong/fortnight	= 1.66309 x 10-4 m/s	- furlong per fortnight
1 in/s, ips	= 0.0254 m/s	- inch per second
1 km/h	= 0.277778 m/s	- kilometer per hour
1 kn, knot	= 0.514444 m/s	- knot (node)
1 cm/s	= 0.01 m/s	- kyne
1 m/min	= 166.667 m/s	- meter per minute
1 m/s	= 1 m/s	- meter per second
1 mpy	= 8.04327 x 10-13 m/s	- mil per year
1 knot	= 0.514444 m/s	- mile (naut.) per hour (knot. node)
1 mph, mi/h	= 0.44704 m/s	- mile (stat.) per hour
1 mi/min	= 26.8224 m/s	- mile (stat.) per minute
1 mm/min	= 1.66667 x 10-5 m/s	- millimeter per minute
1 mm/s	= 0.001 m/s	- millimeter per second

Temperature	
Conversion Type	Conversion Formula
Degrees Celsius (C) to degrees Kelvin (K)	= (C) + 273.15 = (K)
Degrees Celsius (C) to degrees Fahrenheit (F)	= [(C) * 1.8] + 32 = (F)
Degrees Celsius (C) to degrees Rankine (R)	= [(C) * 1.8] + 491.67 = (R)
Degrees Fahrenheit (F) to degrees Kelvin (K)	= [(F) * 0.555556] + 255.37 = (K)
Degrees Fahrenheit (F) to degrees Rankine (R)	= (F) + 459.67 = (R)
Degrees Kelvin (K) to degrees Rankine (R)	= (K) * 1.8 = (R)
Degrees Rankine (R) to degrees Celsius (C)	= [(R) - 491.67] * 0.555556 = (C)
Degrees Rankine (R) to degrees Kelvin (K)	= (R) * 0.555556 = (K)

Pressure		
Symbol	SI equivalent	Designation of unit measure
1 at	= 98066.5 Pa	- atmosphere (metric)
1 atm	= 101325 Pa	- atmosphere (standard)
1 bar	= 100000 Pa	- bar
1 cmHg (0 °C)	= 1333.22 Pa	- centimeter of mercury (0 °C)
1 cmH2O	= 98.0638 Pa	- centimeter of water (4 °C)
1 dyn/cm2	= 0.1 Pa	- dyne per square centimeter
1 ft H2O	= 2988.98 Pa	- foot of water (4 °C)
1 hPa	= 100 Pa	- hectopascal
1 inHg (0 °C)	= 3386.38 Pa	- inch of mercury (0 °C)
1 inHg (15.56 °C)	= 3377.18 Pa	- inch of mercury (15.56 °C)
1 inH2O (15.56 °C)	= 248.845 Pa	- inch of water (15.56 °C)
1 inH2O (4 °C)	= 249.082 Pa	- inch of water (4 °C)
1 kgf/cm2	= 98066.5 Pa	- kilogram force per square centimeter
1 kgf/dm2	= 980.665 Pa	- kilogram force per square decimeter
1 kgf/m2	= 9.80665 Pa	- kilogram force per square meter
1 kgf/mm2	= 9806650 Pa	- kilogram force per square millimeter
1 kPa	= 1000 Pa	- kilopascal
1 kip/in2, ksi, KSI	= 6894760 Pa	- kilopound force per square inch
1 MPa	= 1000000 Pa	- megapascal
1 mH2O, mCE (15.56 °C)	= 9797.06 Pa	- meter of water (15.56 °C)
1 mH2O, mCE (4 °C)	= 9806.38 Pa	- meter of water (4 °C)
1 µbar	= 0.1 Pa	- microbar (barye, barrie)
1 µHg (0 °C)	= 0.133322 Pa	- micron of mercury (millitorr)
1 mbar	= 100 Pa	- millibar
1 mmHg, torr, Torr (0 °C)	= 133.322 Pa	- millimeter of mercury (0 °C)
1 mmH2O, mmCE (15.56 °C)	= 9.79706 Pa	- millimeter of water (15.56 °C)
1 mmH2O, mmCE (4 °C)	= 9.80638 Pa	- millimeter of water (4 °C)
1 N/m2	= 1 Pa	- newton per square meter
1 Pa, N/m2	= 1 Pa	- pascal
1 lbf/ft2	= 47.8803 Pa	- pound force per square foot
1 psi, PSI, lbf/in2	= 6894.76 Pa	- pound force per square inch
1 atm	= 101325 Pa	- standard atmosphere
1 tonf/cm2 (metric)	= 98066500 Pa	- ton force (metric) per square centimeter
1 tonf/m2 (metric)	= 9806.65 Pa	- ton force (metric) per square meter
1 torr	= 133.322 Pa	- torr

Flows rate (Equivalence)		
Symbol	SI equivalent	Designation of unit measure
1 acre-ft/month	= 4.69361 x 10-4 m3/s	- acre foot per month
1 bbl (US, liq.)/day	= 1.3801 x 10-6 m3/s	- barrel (US, liq.) per day
1 bbl (US, petrol)/day	= 1.84013 x 10-6 m3/s	- barrel (US, petrol) per day
1 cm3/s	= 1 x 10-6 m3/s	- cubic centimeter per second
1 CFM, cfm, ft3/min	= 4.71947 x 10-4 m3/s	- cubic foot per minute
1 cfs, ft3/s	= 0.0283168 m3/s	- cubic foot per second
1 in3/min	= 2.73118 x 10-7 m3/s	- cubic inch per minute
1 in3/s	= 1.63871 x 10-5 m3/s	- cubic inch per second
1 m3/h	= 2.77778 x 10-4 m3/s	- cubic meter per hour
1 m3/min	= 0.0166667 m3/s	- cubic meter per minute
1 m3/s	= 1 m3/s	- cubic meter per second
1 yd3/min	= 0.0127426 m3/s	- cubic yard per minute
1 cumec	= 1 m3/s	- cumec (musec)
1 gal (UK)/day	= 5.26168 x 10-8 m3/s	- gallon (UK) per day
1 gph (UK)	= 1.2628 x 10-6 m3/s	- gallon (UK) per hour
1 gpm (UK)	= 7.57682 x 10-5 m3/s	- gallon (UK) per minute
1 gps (UK)	= 4.54609 x 10-3 m3/s	- gallon (UK) per second
1 gal (US, liq.)/day	= 4.38126 x 10-8 m3/s	- gallon (US, liq.) per day
1 gph (US)	= 1.0515 x 10-6 m3/s	- gallon (US, liq.) per hour
1 gpm (US)	= 6.30902 x 10-5 m3/s	- gallon (US, liq.) per minute
1 gps (US)	= 0.00378541 m3/s	- gallon (US, liq.) per second
1 l/h	= 2.77778 x 10-7 m3/s	- liter per hour
1 l/min	= 1.66667 x 10-5 m3/s	- liter per minute
1 l/s	= 0.001 m3/s	- liter per second

Material Constants					
	Density (kg/m^3)	**Elastic Modulus (Pa)**	**Poisson's ratio**	**Thermal expansion coefficient (microns/m C)**	**Heat capacity J/kg(°C)**
Aluminum 2024-T3	2770	7.310E+10	0.3300	22.68	963.00
Aluminum 6061-T6	2700	7.310E+10	0.3300	24.30	963.00
Aluminum 7079-T6	2740	7.172E+10	0.3300	24.66	963.00
Concrete	2242				1000
Copper - pure	8900	1.172E+11		16.56	385.00
Gold - pure	19320	7.448E+10	0.4200	4.39	130.00
Iron	7830				440.00
Lead - pure	11340	1.379E+10	0.4200	52.74	130.00
Magnesium AZ31B-H24	1770	4.483E+10	0.3500	26.10	1047
Magnesium HK31A-H24	1790	4.414E+10	0.3500	25.20	544.00
Nickel - pure	8900	2.207E+11		12.96	461.00
Platinum	21450	1.469E+11	0.3900	9.0000	130.00
Silver - pure	10500	7.241E+10	0.3700	19.80	235.00
Steel AISI 304	8030	1.931E+11	0.2900	17.82	503.00
Steel AISI C1020	7850	2.034E+11	0.2900	11.34	419.00
Tantalum	16600	1.862E+11	0.3500	6.4800	126.00
Titanium B 120VCA	4850	1.021E+11	0.3000	9.3600	544.00
Tungsten	19300	3.448E+11	0.2800	4.5000	138.00

Heat capacity coefficients

		Molar Weight	Heat Capacity					
	Gas	MW [kg/mol]	a [J/(mol K)]	b [J/(mol K^2)]	c [J/(mol K^3)]	d [J/(mol K^4)]	e [J K/mol]	Temp.range [K]
Nitrogen	N_2	0,02801	26,092000	8,2188E-03	-1,9761E-06	1,5927E-10	4,4434E+04	298-6000
Oxygen	O_2	0,032	29,659000	6,1373E-03	-1,1865E-06	9,5780E-11	-2,1966E+05	298-6000
Carbon dioxide	CO_2	0,04401	24,997350	5,5187E-02	-3,3691E-05	7,9484E-09	-1,3664E+05	298-1200
Carbon monoxide	CO	0,02801	25,567590	6,0961E-03	4,0547E-06	-2,6713E-09	1,3102E+05	298-1300
Argon	Ar	0,03995	20,786000	2,8259E-10	-1,4642E-13	1,0921E-17	-3,6614E-02	298-6000
Neon	Ne	0,02018	20,786030	4,8506E-13	-1,5829E-16	1,5251E-20	3,1963E-05	298-6000
Helium	He	0,004	20,786030	4,8506E-13	-1,5929E-16	1,5251E-20	3,1963E-05	298-6000
Hydrogen	H_2	0,00202	33,066178	-1,1363E-02	1,1433E-05	-2,7729E-09	-1,5856E+05	298-1000
Water vapour	H_2O	0,01802	30,092000	6,8325E-03	6,7934E-06	-2,5345E-09	8,2139E+04	500-1700
Ammonia	NH_3	0,01703	19,995630	4,9771E-02	-1,5376E-05	1,9212E-09	1,8917E+05	298-1400
Hydrogen sulphide	H_2S	0,03408	26,884120	1,8678E-02	3,4342E-06	-3,3787E-09	1,3588E+05	298-1400
Benzene	C_6H_6	0,07811	-36,220000	4,8475E-01	-3,1570E-04	7,7620E-08	0,0000E+00	298-1500
Nitrous oxide	N_2O	0,04401	27,679880	5,1149E-02	-3,0645E-05	6,8479E-09	-1,5791E+05	298-1400

Conversion of Unit Systems

Length	Time	Mass	Force	Pressure	Velocity	ρ	Energy	G
m	s	Kg	Kg m/s2	N/m2	m/s	Kg/m3	Kmg2/s2	9.81
m	s	Kg	N	Pa	m/s	m Kg/l	J	9.81
m	s	g	mN	mPa	m/s	μ Kg/l	mJ	9.81
m	s	Mg (ton)	KN	KPa	m/s	Kg/l	KJ	9.81
m	ms	Kg	MN	MPa	Km/s	m Kg/l	MJ	9.81e-6
m	ms	g	KN	KPa	Km/s	μ Kg/l	KJ	9.81e-6
m	ms	Mg (ton)	GN	GPa	Km/s	Kg/l	GJ	9.81e-6
mm	s	Kg	mN	KPa	mm/s	M Kg/l	μ J	9.81e+3
mm	s	g	μ N	Pa	mm/s	K Kg/l	nJ	9.81e+3
mm	s	Mg (ton)	N	MPa	mm/s	G Kg/l	mJ	9.81e+3
mm	ms	Kg	KN	GPa	m/s	M Kg/l	J	9.81e-3
mm	ms	g	N	MPa	m/s	K Kg/l	mJ	9.81e-3
mm	ms	Mg (ton)	MN	TPa	m/s	G Kg/l	KJ	9.81e-3
cm	ms	g	daN	10^5Pa	dam/s	Kg/l	dJ	9.81e-4
				bar				
cm	ms	Kg	10^4 N	10^8 Pa	dam/s	K Kg/l	hJ	9.81e-4
			(KdaN)	(Kbar)				
cm	ms	Mg (ton)	10^7 N	10^{11} Pa	dam/s	M Kg/l	10^5 J	9.81e-4
			(MdaN)	(Mbar)				
cm	μ s	g	10^7 N	10^{11} Pa	10^4 m/s	Kg/l	10^5 J	9.81e-10
			(MdaN)	(Mbar)				

Appendix B
Mïe Grüneisen Equation-of-State

Mïe Grüneisen equation-of-state can be written as:

$$p = \left(K_1\mu + K_2\mu^2 + K_3\mu^3\right)\left(1 - \frac{1}{2}\gamma\mu\right) + \gamma\tilde{E}(1+\mu) \quad \text{EQ. B1}$$

with $\mu = \frac{\rho}{\rho_0} - 1 = \frac{1}{V} - 1$ where V is the relative volume. γ is the gas constant, $\tilde{E}$ the internal energy per initial volume and p the pressure.

The equation can be rewritten in polynomial expression in μ and linear in energy. Let U_s be the shock velocity, C the sound speed and S a material constant. The particle velocity u and shock velocity U_s are related by the following equation:

$$U_S = C + Su \quad \text{EQ. B2}$$

The values of C and S for some materials are given in Table B1. The hydrodynamic shock pressure determined by:

$$p_H = \frac{\rho_0 C^2 x}{(1 - Sx)^2} \quad \text{EQ. B3}$$

with $x = 1 - \frac{\rho_0}{\rho} = 1 - V$. The related energy to the pressure p_H is:

$$E_H = \frac{1}{2} p_H \left(\frac{1}{\rho_0} - \frac{1}{\rho}\right) = \frac{1}{2\rho_0} p_H \; x \quad \text{EQ. B4}$$

The change in pressure with respect to the initial state can be expressed as:

$$p_H - p_0 = \gamma\rho\left(E_H - E_0\right) \quad \text{EQ. B5}$$

$$p - p_0 = \gamma\rho\left(E - E_0\right) \quad \text{EQ. B6}$$

This leads to:

$$p - p_H = \gamma\rho\left(E - E_H\right) \quad \text{EQ. B7}$$

or $p = p_H + \gamma\rho\left(E - \frac{1}{2} p_H \frac{x}{\rho_0}\right) = p_H\left(1 - \frac{\gamma}{2}\frac{\rho x}{\rho_0}\right) + \gamma\rho\, E$ EQ. B8

where $\frac{\rho x}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \mu$ and $\rho\, E = \frac{\rho}{\rho_0}\rho_0 E = (1+\mu)\rho_0 E = (1+\mu)\tilde{E}$. Therefore, the last equation can be written as:

$$p = p_H\left(1-\frac{\gamma}{2}\mu\right)+\gamma(1+\mu)\,\tilde{E} \qquad \text{EQ. B9}$$

The hydrodynamic shock pressure p_H given by EQ. B3 can be expressed in linear polynomial terms:

$$p_H = K_1\mu + K_2\mu^2 + K_3\mu^3 \qquad \text{EQ. B10}$$

with:

$$K_1 = \rho_0 C^2$$

$$K_2 = \rho_0 C^2(2S-1)$$

$$K_3 = \rho_0 C^2(S-1)(3S-1)$$

which allows to obtain the Mïe Grüneisen equation given by EQ. B1.

The equation of state used in RADIOSS is written in a linear polynomial expression:

$$p = c_1\mu + c_2\mu^2 + c_3\mu^3 + (c_4 + c_5\mu)\tilde{E} \qquad \text{EQ. B11}$$

The parameters c_i can be identified by using EQ. B10 and Mïe Grüneisen equation. Under the assumption γ=cst, the following setting is obtained:

$$c_1 = K_1$$

$$c_2 = K_2 - \frac{\gamma}{2}K_1 \qquad \text{EQ. B12}$$

$$c_3 = K_3 - \frac{\gamma}{2}K_2$$

$$c_4 = c_5 = \gamma - 1$$

If γ is not considered to be constant, then:

$$\gamma = \gamma_0 - ax = \gamma_0 - a\left(\frac{\mu}{1+\mu}\right) \qquad \text{EQ. B13}$$

where a is a material characteristic parameter given for some materials in Table B1. The second part of EQ. B9 is expressed as:

$$\gamma(1+\mu)\,\tilde{E} = (\gamma_0 + (\gamma_0 + a)\mu)\tilde{E} \qquad \text{EQ. B14}$$

which leads to:

$$c_1 = K_1$$

$$c_2 = K_2 - \frac{\gamma_0}{2} K_1 = K_1 \left(2S - 1 - \frac{\gamma_0}{2} \right) \quad \text{EQ. B15}$$

$$c_3 = K_3 - \frac{\gamma_0}{2} K_2 + aK_1 \quad \text{EQ. B16}$$

$$c_4 = \gamma_0$$

$$c_5 = \gamma_0 - a$$

Table B1: Equation-of-state parameters

Material	V_0 **(cm³/g)**	**C (cm/ μs)**	**S**	γ_0	**a**
Be	0.540	0.800	1.124	1.16	1.0
Mg(1)	0.562	0.452	1.242	1.63	1.3
Al(2)	0.359	0.533	1.338	2.18	1.7
Ti	0.221	0.470	1.146	1.3	1.1
Stainless(3)	0.1265	0.457	1.490	2.0	1.5
Ni	0.1124	0.465	1.445	2.0	1.5
Cu	0.1118	0.394	1.489	1.97	1.5
Pb	0.0882	0.201	1.54	2.84	2.3

Mg(1): Alloy AZ31B, 96%Mg, 3% Al, 1% Zn.

Al(2): Alloy 2024, 93,5% Al, 4.5% Cu, 1.5% Mg.

Stainless(3):Alloy 304, 72% Fe, 19% Cr, 9% Ni.

Appendix C
Basic Relations of Elasticity

C.1 Isotropic material:

Hooke law 3D (principal stress and strain):

$$\{\sigma\}=[D]\{\varepsilon\}$$

$$\sigma_1 = D_{11}\varepsilon_1 + D_{12}\varepsilon_2 + D_{13}\varepsilon_3$$

$$\sigma_1 = (\lambda + 2\mu)\varepsilon_1 + \lambda(\varepsilon_2 + \varepsilon_3)$$

$$\sigma_1 = \lambda(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + 2\mu\varepsilon_1$$

$$\sigma_1 = K\varepsilon_{kk} + 2\mu e_1 \quad \text{with } \varepsilon_{kk} = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) \text{ and } e_1 = \varepsilon_1 - 1/3(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$$

$$\{\sigma\}=[D]\{\varepsilon\} \ ; \ [D]=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ & 1-\nu & \nu & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 & 0 \\ & & & \frac{1-2\nu}{2} & 0 & 0 \\ & Symm. & & & \frac{1-2\nu}{2} & 0 \\ & & & & & \frac{1-2\nu}{2} \end{bmatrix}$$

$$\{\varepsilon\}=[C]\{\sigma\} \ ; \ [C]=\begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ & & \frac{1}{E} & 0 & 0 & 0 \\ & & & \frac{2(1+\nu)}{E} & 0 & 0 \\ & Symm. & & & \frac{2(1+\nu)}{E} & 0 \\ & & & & & \frac{2(1+\nu)}{E} \end{bmatrix}$$

Hooke law for 2D plan stress:

$$\{\sigma\}=[H]\{\varepsilon\}$$
$$\sigma_1 = H_{11}\varepsilon_1 + H_{12}\varepsilon_2$$

$$\{\sigma\}=[H]\{\varepsilon\} \quad ; \qquad [H]=\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$\{\varepsilon\}=[C]\{\sigma\} \quad ; \qquad [C]=\frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 & 0 & 0 \\ -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix}$$

Hooke law for 2D plan strain:

$$\{\sigma\}=[H]\{\varepsilon\} \quad ; \qquad [H]=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$\{\varepsilon\}=[C]\{\sigma\} \quad ; \qquad [C]=\frac{1+\nu}{E}\begin{bmatrix} 1-\nu & -\nu & 0 \\ -\nu & 1-\nu & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Material Constants Relations

	E, ν	E,G	E,B	G, ν	G,B	B, ν	λ, μ
E	**E**	**E**	**E**	**2(1+v)G**	$\frac{9BG}{3B+G}$	**3(1-2v)B**	$\frac{(3\lambda+2\mu)\mu}{\lambda+\mu}$
$G=\mu$	$\frac{E}{2(1+v)}$	**G**	$\frac{3EB}{9B-E}$	**G**	**G**	$\frac{3(1-2v)B}{2(1+v)}$	μ
B=K	$\frac{E}{3(1-2v)}$	$\frac{EG}{9G-3E}$	**B**	$\frac{2(1+v)G}{3(1-2v)}$	**B**	**B**	$\frac{3\lambda+2\mu}{3}$
ν	ν	$\frac{E-2G}{2G}$	$\frac{3B-E}{6B}$	ν	$\frac{3B-2G}{6B+2G}$	ν	$\frac{\lambda}{2(\lambda+\mu)}$
λ	$\frac{Ev}{(1+v)(1-2v)}$	$\frac{(E-2G)G}{3G-E}$	$\frac{(3B-E)3B}{9B-E}$	$\frac{2Gv}{1-2v}$	$\frac{3B-2G}{3}$	$\frac{3Bv}{(1+v)}$	λ

C.2 Orthotropic material:

General 3D orthotropic case:

The strain-stress relations are defined using 9 material constants:

- Three Young modulus in orthotropic directions 1, 2 and 3 : E_1, E_2, E_3
- Three shear modulus in plans 12, 13 and 23 : G_{12}, G_{13}, G_{23}
- Three Poisson ratios satisfying the relations:

$$\frac{\nu_{12}}{E_1}=\frac{\nu_{21}}{E_2} \quad ; \quad \frac{\nu_{13}}{E_1}=\frac{\nu_{31}}{E_3} \quad ; \quad \frac{\nu_{23}}{E_2}=\frac{\nu_{32}}{E_3}$$

$$1-\nu_{12}\nu_{21}>0 \quad ; \quad 1-\nu_{13}\nu_{31}>0 \quad ; \quad 1-\nu_{23}\nu_{32}>0$$

$$1-\nu_{12}\nu_{21}-\nu_{13}\nu_{31}-\nu_{23}\nu_{32}-\nu_{12}\nu_{23}\nu_{31}-\nu_{21}\nu_{13}\nu_{32}>0$$

$$\{\varepsilon\}=[C]\{\sigma\} \quad ; \quad [C]=\begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{32}}{E} & 0 & 0 & 0 \\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{23}} \end{bmatrix}$$

2D in-plan orthotropic material:

- Orthotropic plan 1-2, isotropic plan 2-3
  - Orthotropy coefficients in the plan 1-2 : $E_1, E_2, \nu_{12}, G_{12}$
  - Isotropy coefficients in plan 2-3: E_2, ν
- Five independent coefficients

$$\{\varepsilon\}=[C]\{\sigma\} \quad ; \qquad [C]=\begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{12}}{E_1} & \frac{-\nu_{12}}{E_1} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu}{E_2} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{-\nu}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{12}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E_2} \end{bmatrix}$$

C.3 Stiffness matrix of beam element:

Terms of the stiffness matrix:

$$[k]=\begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & -K_{11} & 0 & 0 & 0 & 0 & 0 \\ & \frac{12EI_3}{L^3(1+\phi_2)} & 0 & 0 & 0 & \frac{L}{2}K_{22} & 0 & -K_{22} & 0 & 0 & 0 & K_{26} \\ & & \frac{12EI_2}{L^3(1+\phi_2)} & 0 & -\frac{L}{2}K_{33} & 0 & 0 & 0 & -K_{33} & 0 & K_{35} & 0 \\ & & & \frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & -K_{44} & 0 & 0 \\ & & & & \frac{(4+\phi_3)EI_2}{L(1+\phi_3)} & 0 & 0 & 0 & -K_{35} & 0 & \frac{2-\phi_3}{4+\phi_3}K_{55} & 0 \\ & & & & & \frac{(4+\phi_2)EI_3}{L(1+\phi_2)} & 0 & -K_{26} & 0 & 0 & 0 & \frac{2-\phi_2}{4+\phi_2}K_{66} \\ & & & & & & K_{11} & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & K_{22} & 0 & 0 & 0 & -K_{26} \\ & & \mathit{Symm.} & & & & & & K_{33} & 0 & -K_{35} & 0 \\ & & & & & & & & & K_{44} & 0 & 0 \\ & & & & & & & & & & K_{55} & 0 \\ & & & & & & & & & & & K_{66} \end{bmatrix}$$

with : $\phi_2 = \dfrac{144(1+\nu)I_3}{5AL^2}$

$$\phi_3 = \frac{144(1+\nu)I_2}{5AL^2}$$

$$I = \frac{bh^3}{12}$$

for a rectangle cross-section.