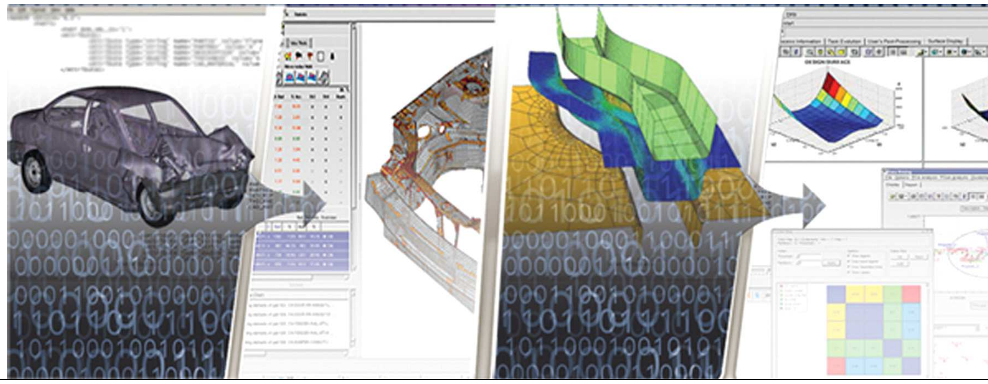

RADIOSS THEORY MANUAL

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Large Displayacement Finite Element Analysis
Chapter 1

Altair

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Chapter 1

INTRODUCTION

1.0 INTRODUCTION

1.1 Introduction

Nonlinear finite element analyses confront users with many choices. An understanding of the fundamental concepts of non linear finite element analysis is necessary if the user does not want to use the finite element program as a black box. The purpose of this manual is to describe the numerical methods included in RADIOSS.

RADIOSS belongs to the family of hydro-codes, in which the material is considered as a non viscous fluid. These hydro-codes found their origin in the work supported by the American Department of Energy at the end of the 70's and which led to software like DYNA2D/3D, HEMP, PRONTO, STEALTH, HONDO and WHAM.

RADIOSS' main features are:

- a 3D Lagrangian formulation for mesh description
- an explicit time integration scheme, leading to small time steps
- simplicity, under integrated finite element models
- element by element assembly of nodal forces leading to in memory codes and low I/O requirements as compared to implicit approaches where matrix assembly and inversion is required every time step
- non-iterative approaches
- penalty methods based contact
- highly vectorized implementation.

This first chapter introduces the notations which will be used throughout the document. An introduction to kinematics is also given.

Chapter 2 recalls the basic equations in non linear dynamics. Different aspects are covered:

- Material and spatial coordinates
- Mesh description
- Kinematic and kinetic descriptions
- Stress rates and stresses in solids
- Updated and total Lagrangian formulations
- Equations of equilibrium
- Principle of virtual power and the physical names of power terms.

The small strain formulation is also introduced.

The finite element formulation of the virtual power principle is introduced in Chapter 3, leading to the discretized equations of equilibrium.

Chapter 4 deals with time discretization and the integration schemes. Stability and time step concepts are also discussed.

Different finite element models are presented in Chapter 5. Tetrahedral solid elements, hexahedral solid and solid-shell elements, 3 and 4-node shell elements, 2-node truss and beam elements and spring elements are successively presented.

Chapter 6 deals with kinematic constraints, i.e. constraints placed on nodal velocities.

Linear stability is introduced in Chapter 7.

The very important concept of interfaces is considered in Chapter 8. Interfaces allow the solution of contact and impact conditions between two parts of a model. The different interface types available in RADIOSS are presented.

Material laws are discussed in Chapter 9.

In Chapter 10, the formulations of different kinds of monitored volumes are presented in detail. Airbag theory is also developed.

Chapter 11 deals with the use of explicit algorithms to model quasi-static or static problems. Different approaches are discussed: slow dynamic computation, dynamic relaxation, viscous relaxation and energy discrete relaxation. The dynamic relaxation approach is developed. The DYREL and DAMP options are introduced in this chapter.

Chapter 12 concerns the presentation of the fundamentals in RADIOSS parallelization.

In the ALE, CFD and SPH Theory Manual, the ALE formulation is presented in Chapter 1.

Finally, Chapters 2 and 3 are respectively dedicated to the Computational Aero-Acoustic and the Smooth Particle Hydrodynamics formulations.

1.2 Notation

Two types of notation are used:

- Indicial notation: Equations of continuum mechanics are usually written in this form.
- Matrix notation: Used for equations pertinent to the finite element implementation.

1.2.1 Index notation

Components of tensors and matrices are given explicitly. A vector, which is a first order tensor, is denoted in indicial notation by x_i. The range of the index is the dimension of the vector.

To avoid confusion with nodal values, coordinates will be written as *x*, *y* or *z* rather than using subscripts. Similarly, for a vector such as the velocity v_i, numerical subscripts are avoided so as to avoid confusion with node numbers. So, $x_1 = x, \quad x_2 = y, \quad x_3 = z$ and $v_1 = v_x, \quad v_2 = v_y$ and $v_3 = v_2$.

Indices repeated twice in a list are summed. Indices which refer to components of tensors are always written in lower case. Nodal indices are always indicated by upper case Latin letters. For instance, v_{iI} is the i-component of the velocity vector at node I. Upper case indices repeated twice are summed over their range.

A second order tensor is indicated by two subscripts. For example, E_{ij} is a second order tensor whose components are $E_{xx}, E_{xy}, ...$

1.2.2 Matrix notation

Matrix notation is used in the implementation of finite element models. For instance, equation

$$r^2 = x_i \cdot x_i = x_1 \cdot x_1 + x_2 \cdot x_2 + x_3 \cdot x_3 \qquad \text{EQ. 1.2.2.1}$$

is written in matrix notation as:

$$r^2 = x^T x \qquad \text{EQ. 1.2.2.2}$$

All vectors such as the velocity vector *v* will be denoted by lower case letters. Rectangular matrices will be denoted by upper case letters.

1.3 Kinematics of particles

Kinematics deals with position in space as a function of time and is often referred to as the “geometry of motion” [96]. The motion of particles may be described through the specification of both linear and angular coordinates and their time derivatives. Particle motion on straight lines is termed *rectilinear* motion, whereas motion on curved paths is called *curvilinear* motion. Although the rectilinear motion of particles and rigid bodies is well-known and used by engineers, the space curvilinear motion needs some feed-back, which is described in the following section. The reader is invited to consult [96] for more details.

1.3.1 Space curvilinear motion

The motion of a particle along a curved path in space is called *space curvilinear* motion. The position vector ***R***, the velocity ***v***, and the acceleration of a particle along a curve are:

$$\mathrm{R} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}} \qquad \text{EQ. 1.3.1.1}$$

$$\mathrm{v} = \dot{\mathrm{R}} = \dot{x}\hat{\mathrm{i}} + \dot{y}\hat{\mathrm{j}} + \dot{z}\hat{\mathrm{k}} \qquad \text{EQ. 1.3.1.2}$$

$$\mathrm{a} = \ddot{\mathrm{R}} = \ddot{x}\hat{\mathrm{i}} + \ddot{y}\hat{\mathrm{j}} + \ddot{z}\hat{\mathrm{k}} \qquad \text{EQ. 1.3.1.3}$$

where x, y and z are the coordinates of the particle and $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$ and $\hat{\mathrm{k}}$ the unit vectors in the rectangular reference. In the cylindrical reference (r, θ, z), the description of space motion calls merely for the polar coordinate expression:

$$\mathrm{v} = \mathrm{v}_\mathrm{r} + \mathrm{v}_\theta + \mathrm{v}_\mathrm{z} \qquad \text{EQ. 1.3.1.4}$$

where:

$$v_r = \dot{r}$$

$$v_\theta = r\dot{\theta}$$

$$v_z = \dot{z}$$

Also, for acceleration:

$$\mathrm{a} = \mathrm{a}_\mathrm{r} + \mathrm{a}_\theta + \mathrm{a}_\mathrm{z} \qquad \text{EQ. 1.3.1.5}$$

where:

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$a_z = \ddot{z}$$

The vector location of a particle may also be described by spherical coordinates as shown in Figure 1.3.1.

$$\mathrm{v} = \mathrm{v}_\mathrm{R} + \mathrm{v}_\theta + \mathrm{v}_\phi \qquad \text{EQ. 1.3.1.6}$$

where:

$$v_R = \dot{R}$$

$$v_\theta = R\dot{\theta}\cos\phi$$

$$v_\phi = R\dot{\phi}$$

Using the previous expressions, the acceleration and its components can be computed:

$$a = a_R + a_\theta + a_\phi \qquad \text{EQ. 1.3.1.7}$$

where:

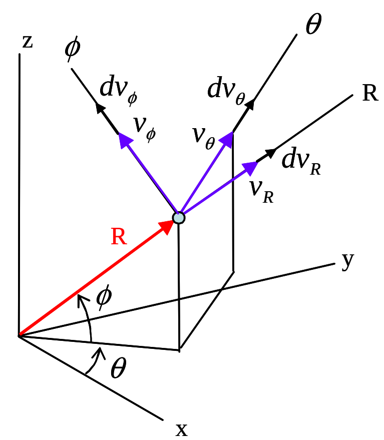
$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2 \cos^2\phi$$

$$a_\theta = \frac{\cos\phi}{R}\frac{d}{dt}\left(R^2\dot{\theta}\right) - 2R\dot{\theta}\dot{\phi}\sin\phi$$

$$a_\phi = \frac{1}{R}\frac{d}{dt}\left(R^2\dot{\phi}\right) + R\dot{\theta}^2 \sin\phi\cos\phi$$

The choice of the coordinate system simplifies the measurement and the understanding of the problem.

Figure 1.3.1 Vector location of a particle in rectangular, cylindrical and spherical coordinates



1.3.2 Coordinate transformation

It is frequently necessary to transform vector quantities from a given reference to another. This transformation may be accomplished with the aid of matrix algebra. The quantities to transform might be the velocity or acceleration of a particle. It could be its momentum or merely its position, considering the transformation of a velocity vector when changing from rectangular to cylindrical coordinates:

$$\begin{Bmatrix} V_r \\ V_\theta \\ V_z \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_x \\ V_y \\ V_z \end{Bmatrix} \quad \text{or} \quad \{V_{r\theta z}\} = [T_\theta]\{V_{xyz}\} \qquad \text{EQ. 1.3.2.1}$$

The change from cylindrical to spherical coordinates is accomplished by a single rotation ϕ of the axes around the θ-axis. The transfer matrix can be written directly from the previous equation where the rotation ϕ occurs in the *R*-ϕ plane:

$$\begin{Bmatrix} V_R \\ V_\theta \\ V_\phi \end{Bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{Bmatrix} V_r \\ V_\theta \\ V_z \end{Bmatrix} \quad \text{or} \quad \{V_{R\theta\phi}\} = [T_\phi]\{V_{r\theta z}\} \qquad \text{EQ. 1.3.2.2}$$

Direct transfer from rectangular to spherical coordinates may be accomplished by combining EQ. 1.3.2.1 and EQ. 1.3.2.2:

$$\{V_{R\theta\phi}\} = [T_\phi][T_\theta]\{V_{xyz}\} \qquad \text{EQ. 1.3.2.3}$$

with: $$[T_\phi][T_\theta] = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \\ -\sin\theta & \cos\theta & 0 \\ -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \end{bmatrix}$$

1.3.3 Transformation of reference axes

Consider now the curvilinear motion of two particles A and B in space. Let's study at first the translation of a reference without rotation. The motion of A is observed from a translating frame of reference x-y-z moving with the origin B (Figure 1.3.2). The position vector of A relative to B is:

$$\mathrm{r}_{A/B} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + \mathrm{z}\hat{\mathrm{k}} \qquad \text{EQ. 1.3.3.1}$$

where $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$ and $\hat{\mathrm{k}}$ are the unit vectors in the moving x-y-z system. As there is no change of unit vectors in time, the velocity and the acceleration are derived as:

$$\mathrm{v}_{A/B} = \dot{x}\hat{\mathrm{i}} + \dot{y}\hat{\mathrm{j}} + \dot{\mathrm{z}}\hat{\mathrm{k}} \qquad \text{EQ. 1.3.3.2}$$

$$\mathrm{a}_{A/B} = \ddot{x}\hat{\mathrm{i}} + \ddot{y}\hat{\mathrm{j}} + \ddot{\mathrm{z}}\hat{\mathrm{k}} \qquad \text{EQ. 1.3.3.3}$$

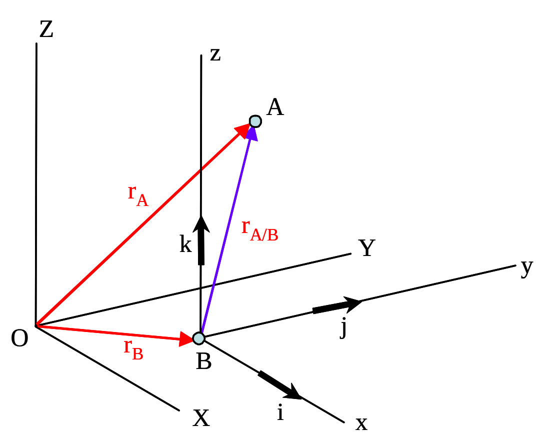
The absolute position, velocity and acceleration of A are then:

$$\mathrm{r_A} = \mathrm{r_B} + \mathrm{r_{A/B}} \qquad \text{EQ. 1.3.3.4}$$

$$\mathrm{v_A} = \mathrm{v_B} + \mathrm{v_{A/B}}$$

$$\mathrm{a_A} = \mathrm{a_B} + \mathrm{a_{A/B}}$$

Figure 1.3.2 Vector location with a moving reference



In the case of rotation reference, it is proved that the angular velocity of the reference axes x-y-z may be represented by the vector:

$$\omega = \omega_x \, \hat{\mathrm{i}} + \omega_y \, \hat{\mathrm{j}} + \omega_z \, \hat{\mathrm{k}} \qquad \text{EQ. 1.3.3.5}$$

The time derivatives of the unit vectors $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$ and $\hat{\mathrm{k}}$ due to the rotation of reference axes x-y-z about ω, can be studied by applying an infinitesimal rotation ωdt . We can write:

$$\frac{d}{dt}\left(\hat{\mathrm{i}}\right) = \omega \times \hat{\mathrm{i}} \quad ; \quad \frac{d}{dt}\left(\hat{\mathrm{j}}\right) = \omega \times \hat{\mathrm{j}} \quad ; \quad \frac{d}{dt}\left(\hat{\mathrm{k}}\right) = \omega \times \hat{\mathrm{k}} \qquad \text{EQ. 1.3.3.6}$$

Attention should be turned to the meaning of the time derivatives of any vector quantity **V**=V_x**i** + V_y**j** + V_z**k** in the rotating system. The derivative of **V** with respect to time as measured in the fixed frame ***X-Y-Z*** is:

$$\left(\frac{d\mathrm{V}}{dt}\right)_{XYZ} = \frac{d}{dt}\left(V_x \hat{\mathrm{i}} + V_y \hat{\mathrm{j}} + V_z \hat{\mathrm{k}}\right) \qquad \text{EQ. 1.3.3.7}$$

$$= \left(V_x \frac{d}{dt}\left(\hat{\mathrm{i}}\right) + V_y \frac{d}{dt}\left(\hat{\mathrm{j}}\right) + V_z \frac{d}{dt}\left(\hat{\mathrm{k}}\right)\right) + \left(\dot{V}_x \hat{\mathrm{i}} + \dot{V}_y \hat{\mathrm{j}} + \dot{V}_z \hat{\mathrm{k}}\right)$$

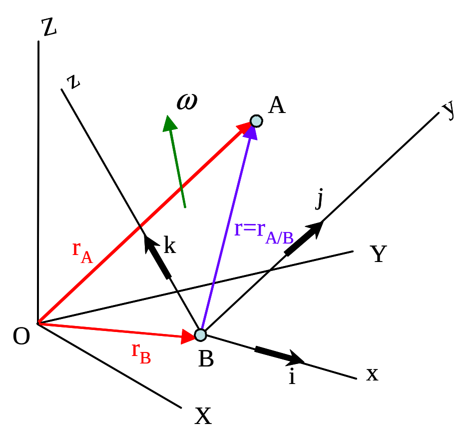
With the substitution of EQ. 1.3.3.6, the terms in the first parentheses becomes $\omega \times \mathrm{V}$. The terms in the second parentheses represent the components of time derivatives $\left(\frac{d\mathrm{V}}{dt}\right)_{xyz}$ as measured relative to the moving x-y-z reference axes. Thus:

$$\left(\frac{d\mathrm{V}}{dt}\right)_{XYZ} = \omega \times \mathrm{V} + \left(\frac{d\mathrm{V}}{dt}\right)_{xyz} \qquad \text{EQ. 1.3.3.8}$$

This equation establishes the relation between the time derivative of a vector quantity in a fixed system and the time derivative of the vector as observed in the rotating system.

Consider now the space motion of a particle *A*, as observed both from a rotating system x-y-z and a fixed system X-Y-Z (Figure 1.3.3).

Figure 1.3.3 Vector location with a rotating reference



The origin of the rotating system coincides with the position of a second reference particle *B*, and the system has an angular velocity ω. Standing **r** for $r_{A/B}$, the time derivative of the vector position gives:

$$r_A = r_B + r \quad \Rightarrow \quad v_A = v_B + \dot{r} \qquad \text{EQ. 1.3.3.9}$$

From EQ. 1.3.3.8:

$$\dot{r} = \left(\frac{dr}{dt}\right)_{XYZ} = \omega \times r + v_{rel} \qquad \text{EQ. 1.3.3.10}$$

where V_{rel} denotes the relative velocity measured in x-y-z, i.e.:

$$v_{rel} = (dr/dt)_{xyz} = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k} \qquad \text{EQ. 1.3.3.11}$$

Thus the relative velocity equation becomes:

$$\boxed{v_A = v_B + \omega \times r + v_{rel}} \qquad \text{EQ. 1.3.3.12}$$

The relative acceleration equation is the time derivative of EQ. 1.3.3.12 which gives:

$$a_A = a_B + \dot{\omega} \times r + \omega \times \dot{r} + \dot{v}_{rel} \qquad \text{EQ. 1.3.3.13}$$

where the last term can be obtained from EQ. 1.3.3.8:

$$\dot{v}_{rel} = \left(\frac{dv_{rel}}{dt}\right)_{XYZ} = \omega \times v_{rel} + a_{rel} \qquad \text{EQ. 1.3.3.14}$$

and $$a_{rel} = \left(\frac{dv_{rel}}{dt}\right)_{xyz} = \ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} + \ddot{z}\,\hat{k} \qquad \text{EQ. 1.3.3.15}$$

Combining EQ. 1.3.3.13 to 15, we obtain upon collection of terms:

$$\boxed{a_A = a_B + \dot{\omega} \times r + \omega \times (\omega \times r) + 2\omega \times v_{rel} + a_{rel}} \qquad \text{EQ. 1.3.3.16}$$

where the term $2\omega \times v_{rel}$ constitutes Coriolis acceleration.

1.3.4 Skew and Frame notions in RADIOSS

Two kinds of reference definition are available in RADIOSS:

- Skew reference:

It is a projection reference to define the local quantities with respect to the global reference. In fact the origin of skew remains at the initial position during the motion even though a moving skew is defined. In this case, a simple projection matrix is used to compute the kinematic quantities in the reference.

- Frame reference:

It is a mobile or fixed reference. The quantities are computed with respect to the origin of the frame which may be in motion or not depending to the kind of reference frame. For a moving reference frame, the position and the orientation of the reference vary in time during the motion. The origin of the frame defined by a node position is tied to the node. EQ. 1.3.3.12 and 16 are used to compute the accelerations and velocities in the frame.