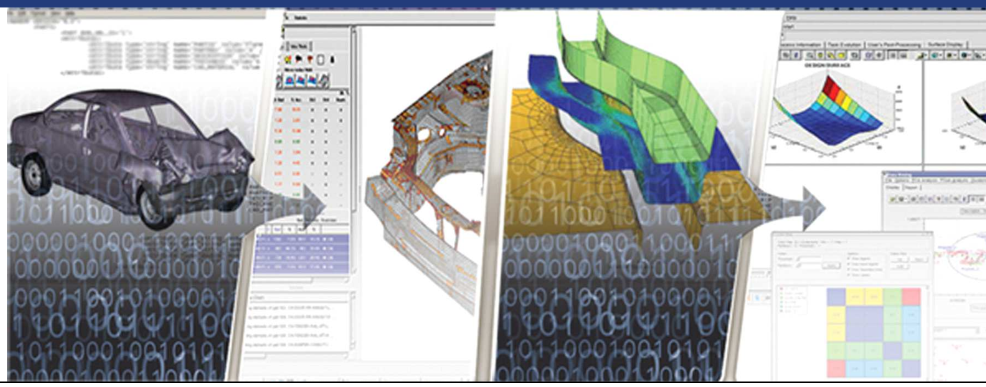

RADIOSS THEORY MANUAL

Version 2017 – January 2017

Large Displacement Finite Element Analysis

Chapter 6

Altair

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Chapter 6

KINEMATIC CONSTRAINTS

6.0 KINEMATIC CONSTRAINTS

Kinematic constraints are boundary conditions that are placed on nodal velocities. They are mutually exclusive for each degree of freedom (DOF), and there can only be one constraint per DOF.

There are seven different types of kinematic constraints that can be applied to a model in RADIOSS:

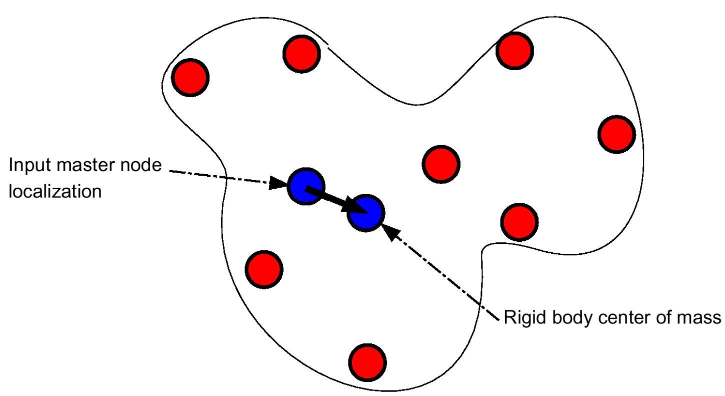
1. Rigid Body
2. Initial static equilibrium
3. Boundary Condition
4. Tied Interface (Type 2)
5. Rigid Wall
6. Rigid Link
7. Cylindrical Joint

Two kinematic conditions applied to the same node may be incompatible.

6.1 Rigid Body

A rigid body is defined by a master node and its associated slave nodes. Mass and inertia may be added to the initial master node location. The master node is then moved to the center of mass, taking into account the master node and all slave node masses. Figure 6.1.1 shows an idealized rigid body.

Figure 6.1.1 Idealized Rigid Body



6.1.1 Rigid body mass

The mass of the rigid body is calculated by:

$$m = m^M + \sum_I m^I \qquad \text{EQ. 6.1.1.1}$$

The rigid body's center of mass is defined by:

$$x^G = \frac{m^M x^M + \sum m^I x^I}{m} \qquad \text{EQ. 6.1.1.2}$$

$$y^G = \frac{m^M y^M + \sum m^I y^I}{m}$$ EQ. 6.1.1.3

$$z^G = \frac{m^M z^M + \sum m^I z^I}{m}$$ EQ. 6.1.1.4

where

m^M is the master node mass

m^I are the slave node masses

x^G, y^G, z^G are the coordinates of the mass center.

6.1.2 Rigid body inertia

The six components of inertia of a rigid body are computed by:

$$I_{xx} = J_{xx}^M + m^M \left((y_M - y_G)^2 + (z_M - z_G)^2 \right) + \sum_i I_{xx}^i + m^i \left((y_i - y_G)^2 + (z_i - z_G)^2 \right)$$ EQ. 6.1.2.1

$$I_{yy} = J_{yy}^M + m^M \left((x_M - x_G)^2 + (z_M - z_G)^2 \right) + \sum_i I_{yy}^i + m^i \left((x_i - x_G)^2 + (z_i - z_G)^2 \right)$$ EQ. 6.1.2.2

$$I_{zz} = J_{zz}^M + m^M \left((x_M - x_G)^2 + (y_M - y_G)^2 \right) + \sum_i I_{zz}^i + m^i \left((x_i - x_G)^2 + (y_i - y_G)^2 \right)$$ EQ. 6.1.2.3

$$I_{xy} = J_{xy}^M + m^M \left((x_M - x_G) + (y_M - y_G) \right) + \sum_i I_{xy}^i - m^i \left((x_i - x_G) + (y_i - y_G) \right)$$ EQ. 6.1.2.4

$$I_{yz} = J_{yz}^M + m^M \left((y_M - y_G) + (z_M - z_G) \right) + \sum_i I_{yz}^i - m^i \left((y_i - y_G) + (z_i - z_G) \right)$$ EQ. 6.1.2.5

$$I_{xz} = J_{xz}^M + m^M \left((x_M - x_G) + (z_M - z_G) \right) + \sum_i I_{xz}^i - m^i \left((x_i - x_G) + (z_i - z_G) \right)$$ EQ. 6.1.2.6

where I_{ij} is the moment of rotational inertia in the *ij* direction. J_{ij}^M is the master node added inertia.

6.1.3 Rigid body force and moment computation

The forces and moments acting on the rigid body are calculated by:

$$\vec{F} = \vec{F}^M + \sum_i \vec{F}^i \quad \text{EQ. 6.1.3.1}$$

$$\vec{M} = \vec{M}^M + \sum_i \vec{M}^i + \sum_i S_i \vec{G} \times \vec{F}^i \quad \text{EQ. 6.1.3.2}$$

where

$\vec{F}^M$ is the force vector at the master node

$\vec{F}^i$ is the force vector at the slave nodes

$\vec{M}^M$ is the moment vector at the master node

$\vec{M}^i$ is the moment vector at the slave nodes

$\vec{G}$ is the vector from slave node to the center of mass

Resolving these into orthogonal components, the linear and rotational acceleration may be computed as:

Linear Acceleration

$$\gamma_i = \frac{F_i}{m} \quad \text{EQ. 6.1.3.3}$$

Rotational Acceleration

$$I_1 \alpha_1 = M_1 - (I_3 - I_2)\omega_2 \omega_3 \quad \text{EQ. 6.1.3.4}$$

$$I_2 \alpha_2 = M_2 - (I_1 - I_3)\omega_1 \omega_3 \quad \text{EQ. 6.1.3.5}$$

$$I_3 \alpha_3 = M_3 - (I_2 - I_1)\omega_1 \omega_2 \quad \text{EQ. 6.1.3.6}$$

where

I_i are the principal moments of inertia of the rigid body

α_i are the rotational accelerations in the principal inertia frame (reference frame)

ω_i is the rotational velocity in the principal inertia frame (reference frame)

M_i are the moments in the principal inertia frame (reference frame)

6.1.4 Time integration

Time integration is performed to find velocities of the rigid body at the master node:

$$\vec{v}\left(t+\frac{\Delta t}{2}\right)=\vec{v}\left(t-\frac{\Delta t}{2}\right)+\vec{\gamma}(t)\Delta t \qquad \text{EQ. 6.1.4.1}$$

$$\vec{\omega}\left(t+\frac{\Delta t}{2}\right)=\vec{\omega}\left(t-\frac{\Delta t}{2}\right)+\vec{\alpha}(t)\Delta t \qquad \text{EQ. 6.1.4.2}$$

where $\vec{v}$ is the linear velocity vector. Rotational velocities are computed in the local reference frame.

The velocities of slave nodes are computed by:

$$\vec{v}^{\,i}=\vec{v}^{\,M}+S_i\vec{G}x\vec{\omega} \qquad \text{EQ. 6.1.4.3}$$

$$\vec{\omega}^{i}=\vec{\omega}^{M} \qquad \text{EQ. 6.1.4.4}$$

6.1.5 Rigid body boundary conditions

The boundary conditions given to slave nodes are ignored. The rigid body has the boundary conditions given to the master node only.

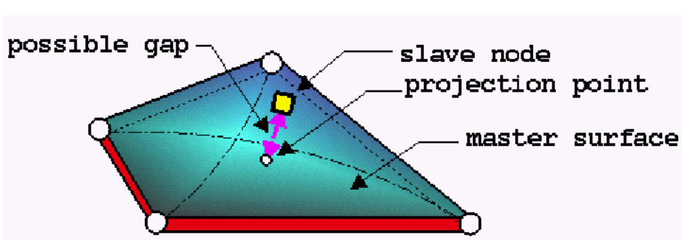
A kinematic condition is applied on each slave node, for all directions. A slave node is not allowed to have any other kinematic conditions.

No kinematic condition is applied on the master node. However, the rotational velocities are computed in a local reference frame. This reference frame is not compatible with all options imposing rotation such as imposed velocity, rotational, rigid link.

The only exception concerns the rotational boundary conditions for which a special treatment is carried out. Connecting shell, beam or spring with rotation stiffness to the master node, is not yet allowed either.

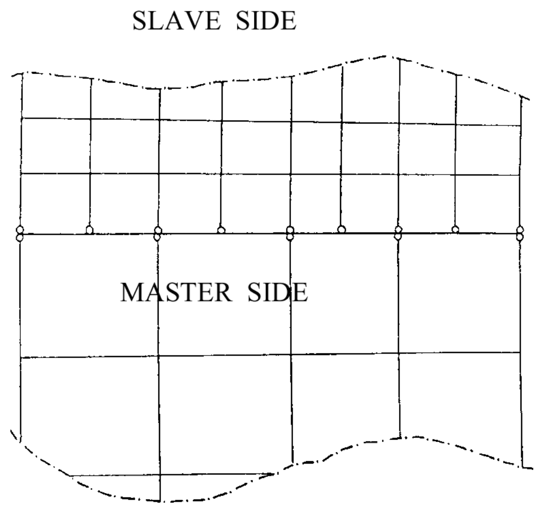
6.2 Tied Interface (Type 2)

With a tied interface it is possible to connect rigidly a set of slave nodes to a master surface.



A tied interface (Type 2) can be used to connect a fine mesh of Lagrangian elements to a coarse mesh or two different kinds of meshes (for example spring to shell contacts).

Figure 6.2.1 Fine and coarse mesh



A master and a slave surface are defined in the interface input cards. The contact between the two surfaces is tied. No sliding or movement of the slave nodes is allowed on the master surface. There are no voids present either.

It is recommended that the master surface has a coarser mesh.

Accelerations and velocities of the master nodes are computed with forces and masses added from the slave nodes.

Kinematic constraint is applied on all slave nodes. They remain at the same position on their master segments.

Tied interfaces are useful in rivet modeling, where they are used to connect springs to a shell or solid mesh.

6.2.1 Spotweld formulation

The slave node is rigidly connected to the master surface. Two formulations are available to describe this connection:

- Default formulation
- Optimized formulation

6.2.1.1 Default spotweld formulation

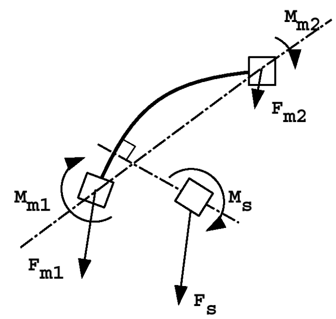
SPOTflag=0

When the flag is set to 0, the spotweld formulation is a default formulation:

- based on element shape functions.
- generating hourglass with under integrated elements.
- providing a connection stiffness function of slave node localization.
- recommended with full integrated shells (master).
- recommended for connecting brick slave nodes to brick master segments (mesh transition without rotational freedom).

Forces and moments transfer from slave to master nodes is described in Figure 6.2.2. :

Figure 6.2.2 Default tied Interface (Type 2)

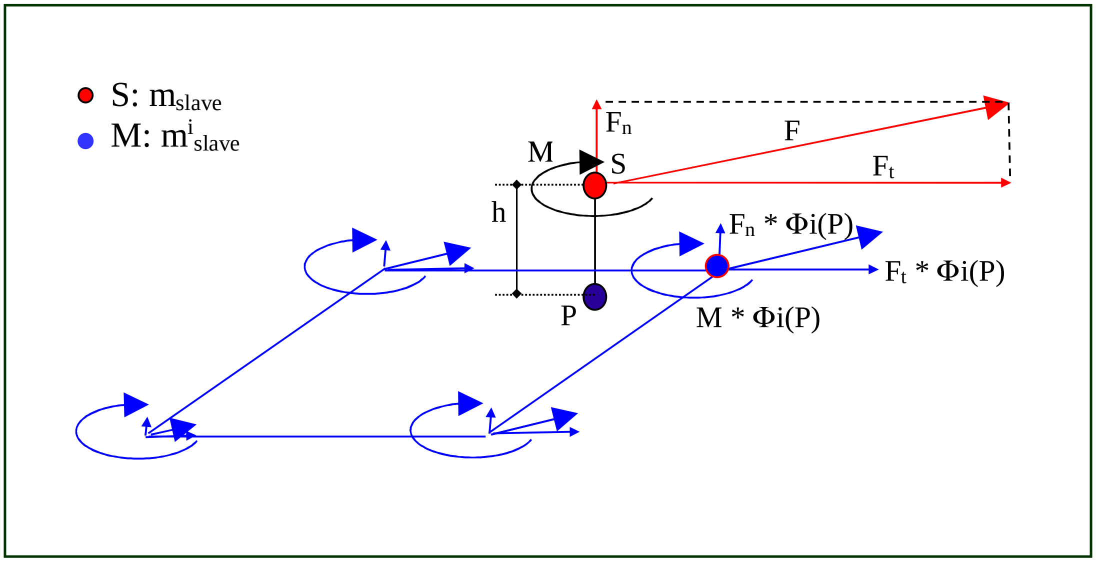


The mass of the slave node is transferred to the master nodes using the position of the projection on the segment and linear interpolation functions:

$$\overline{m}_{master}^{i} = m_{master}^{i} + m_{slave} * \Phi_i(\mathrm{p}) \qquad \text{EQ. 6.2.1.1}$$

where p denotes the position of the slave point and Φ is the weight function obtained by the interpolation equations.

Figure 6.2.3 Transfer of slave node efforts to the master nodes (SPOTflag=0)



The inertia of the slave node is also transferred to the master nodes by taking into account the distance *d* between the slave node and the mater surface:

$$\overline{I}_{master}^{i} = I_{master}^{i} + \left(I_{slave} + m_{slave} * d^2\right) * \Phi_i(\mathrm{p}) \qquad \text{EQ. 6.2.1.2}$$

The term $m_{slave} * d^2$ may increase the total inertia of the model especially when the slave node is far from the master surface.

The stability conditions are written on the master nodes:

$$\overline{K}_{master} = K_{master} + K_{slave} * \Phi_i(\mathrm{p}) \qquad \text{EQ. 6.2.1.3}$$

$$\overline{K}_{master}^{rotation} = K_{master}^{rotation} + \left(K_{slave}^{rotation} + K_{slave} * d^2\right) * \Phi_i(\mathrm{p})$$

The dynamic equilibrium of each master node is then studied and the nodal accelerations are computed. Then the velocities at master nodes can be obtained and updated to compute the velocity of the projected point *P* by the following expressions:

$$V_P^{translation} = \sum_i V_{master\ i}^{translation} \Phi_i(\mathrm{p}) \qquad \text{EQ. 6.2.1.4}$$

$$V_P^{rotation} = \sum_i V_{master\ i}^{rotation} \Phi_i(\mathrm{p}) \qquad \text{EQ. 6.2.1.4}$$

The velocity of the slave node is then obtained:

$$V_{slave}^{translation} = V_P^{translation} + V_P^{rotation} \otimes \overrightarrow{PS} \qquad \text{EQ. 6.2.1.5}$$

$$V_{slave}^{rotation} = V_P^{rotation}$$

With this formulation, the added inertia may be very large especially when the slave node is far from the mean plan of the master element.

6.2.1.2 Optimized spotweld formulation

SPOTflag=1

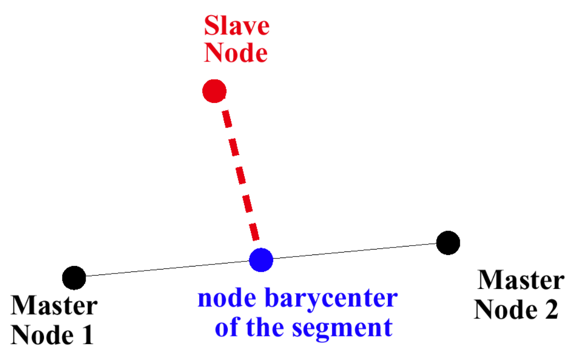
When the flag is set to 1, the spotweld formulation is an optimized formulation:

- based on element mean rigid motion (i.e. without exciting deformation modes)
- having no hourglass problem
- having constant connection stiffness
- recommended with under integrated shells (master)
- recommended for connecting beam, spring and shell slave nodes to brick master segments

This spotweld formulation is optimized for spotwelds or rivets.

The slave node is joined to the master segment barycenter as shown in Figure 6.2.4.

Figure 6.2.4 Relation between slave node and master node



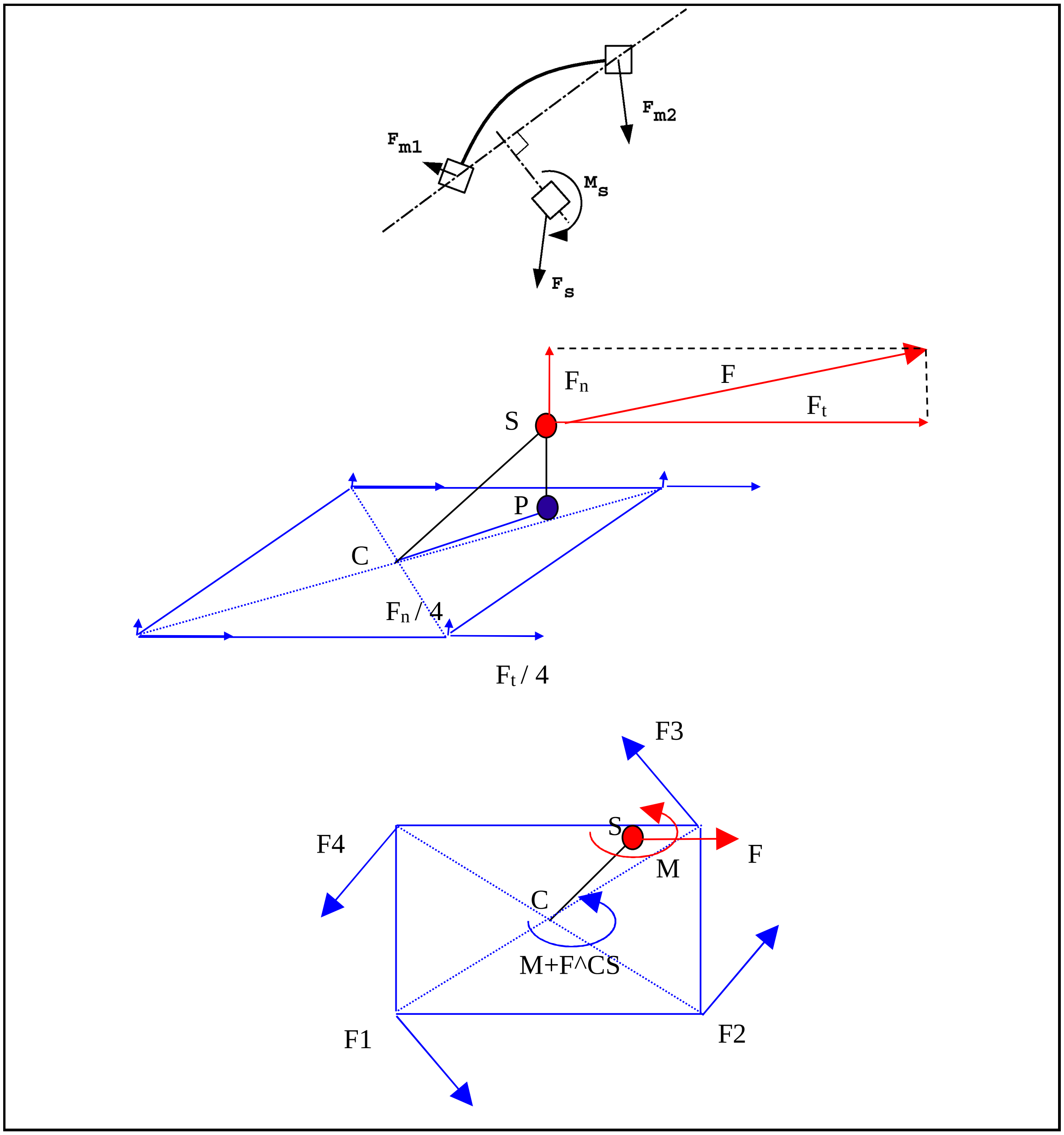
Forces and moments transfer from slave to master nodes is described in Figure 6.2.5. The force applied at the slave node *S* is redistributed uniformly to the master nodes. In this way, only translational mode is excited. The moment $M + CS \otimes F$ is redistributed to the master nodes by four forces F_i such that:

$$F_i \propto \vec{A} \otimes CM_i \qquad \text{EQ. 6.2.1.6}$$

$$\sum_i CM_i \otimes F_i = M + CS \otimes F$$

where $\vec{A}$ is the normal vector to the segment.

Figure 6.2.5 Optimized tied Interface (Type 2)



In this formulation the mass of the slave node is equally distributed to the master nodes. In conformity with effort transmission as described in 6.2.1.2, the spherical inertia is computed with respect to the center of the master element C:

$$I_C^{Slave} = I^{Slave} + m^{Slave}.d^2 \qquad \text{EQ. 6.2.1.7}$$

where *d* is distance from the slave node to the center of element. In order to insure the stability condition without reduction in the time step, the inertia of the slave node is transferred to the master nodes by an equivalent nodal mass computed by:

$$\Delta m = \frac{I^{Slave} + \mathrm{m}^{Slave}.d^2}{\left\| \bar{I} \right\|}, \quad with \quad \bar{I} = \sum_{i=1,..4} \begin{pmatrix} Y_i^2 + Z_i^2 & -X_iY_i & -X_iZ_i \\ -X_iY_i & X_i^2 + Z_i^2 & -Y_iZ_i \\ -X_iZ_i & -Y_iZ_i & X_i^2 + Y_i^2 \end{pmatrix} \qquad \text{EQ. 6.2.1.8}$$

For this reason the formulation causes an increase of mass which may become very important especially when the node is far from the mean surface of the master shell element.

6.2.2 Formulation for search of closest master segment

The master segment is found via 2 formulations:

- Old formulation
- New improved formulation

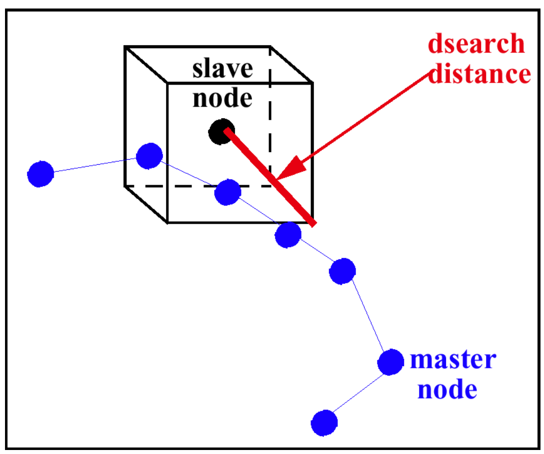
6.2.2.1 Old search of closest master segment formulation

Isearch=1

When the flag is set to 1, the search of closest master segment was based on the old formulation:

A box with a side equal to dsearch (input) is built to search the master node contained within this box.

Figure 6.2.6 Old search of closest master segment

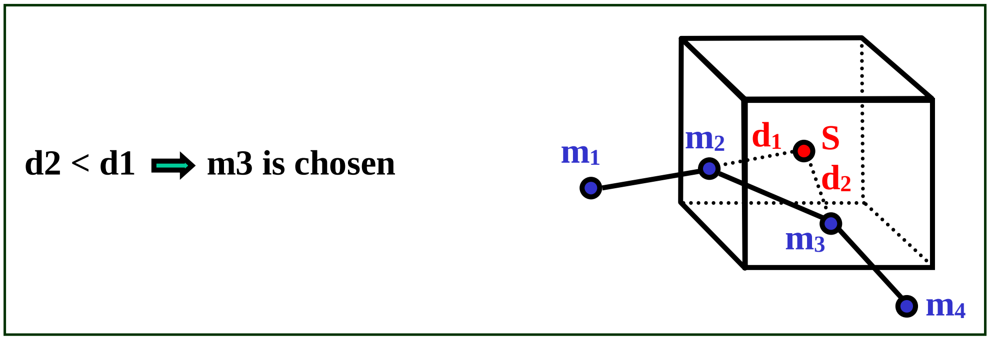


The distance between each master node in the box and the slave node is computed.

The master node giving the minimum distance (dmin) is retained.

The segment is chosen with the selected node, (if the selected node belongs to 2 segments, one is selected at random).

Figure 6.2.7 Old search of closest master segment



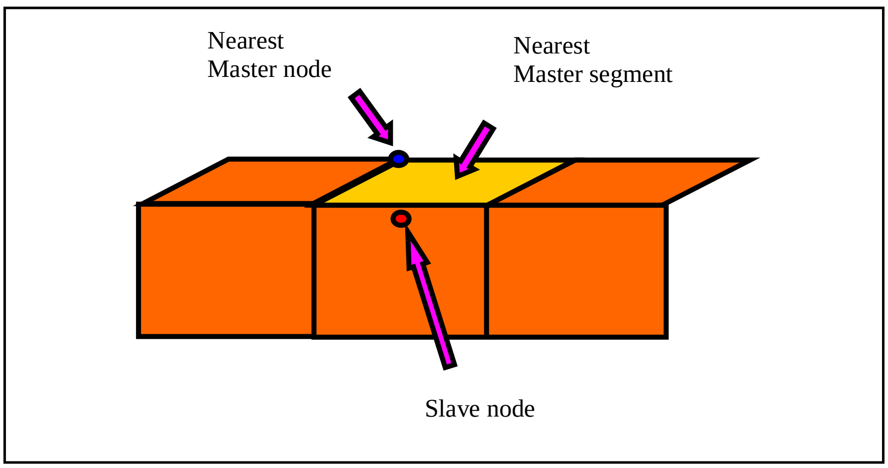
6.2.2.2 New improved search of closest master segment formulation

Isearch=2

When the flag is set to 2, the search of closest master segment is based on the new improved formulation; a box including the master surface is built.

The dichotomy principle is applied to this box as long as the box contains only one master node and as long as the box side is equal to dsearch.

Figure 6.2.8 New improved search of closest master segment

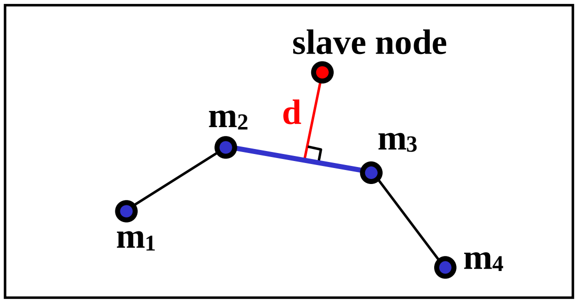


There are two solutions to compute the minimum distance, dmin:

1- The slave node is an internal node for the master segment, as shown in Figure 6.2.9.

The slave node is projected orthogonally on the master segment to give a distance that may be compared with other distances. Select the minimum distance:

Figure 6.2.9 Orthogonal projection on the master segment

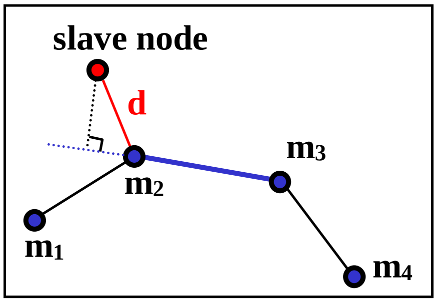


The segment that provides the minimum distance is chosen for the following computation.

2- The slave node is a node external to the master segment, as shown in Figure 6.2.10.

The distance selected is that between the slave node and the nearest master node.

Figure 6.2.10 . Nearest master node



The segment is chosen using the selected node, (if the selected node belongs to 2 segments, one is chosen at random).

6.3 Rigid Wall

There are four types of rigid walls available in RADIOSS:

1. Infinite Plane
2. Infinite Cylinder with Diameter D
3. Sphere with Diameter D
4. Parallelogram

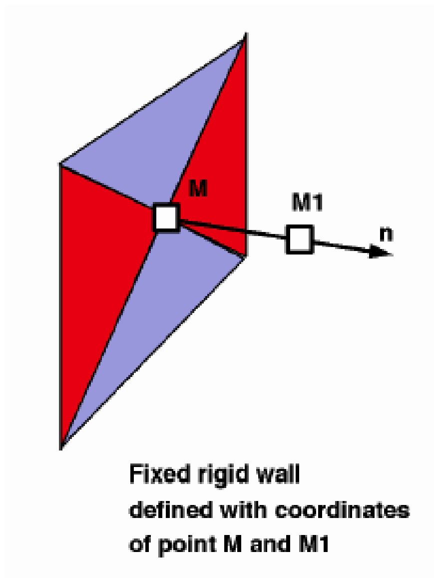
Each wall can be fixed or moving.

A kinematic condition is applied on each impacted slave node. Therefore, a slave node cannot have another kinematic condition; unless these conditions are applied in orthogonal directions.

6.3.1 Fixed rigid wall

A fixed wall is a pure kinematic option on all impacted slave nodes. It is defined using two points, M and M1. These define the normal, as shown in Figure 6.3.1.

Figure 6.3.1 Fixed Rigid Wall Definition

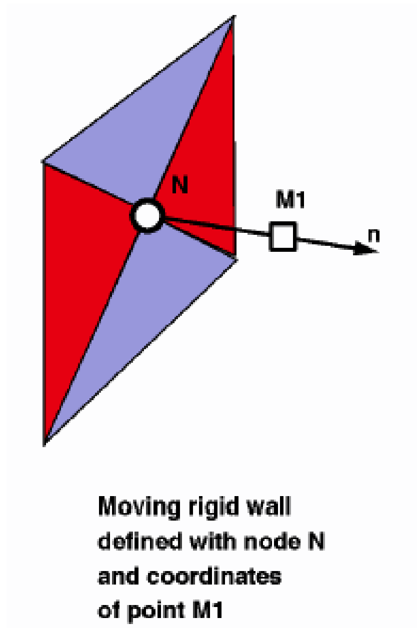


6.3.2 Moving rigid wall

A moving rigid wall is defined by a node number, N, and a point, M1. This allows a normal to be calculated, as shown in Figure 6.3.2.

The motion of node N can be specified with fixed velocity, or with an initial velocity. For simplification, an initial velocity and a mass may be given at the wall definition level.

Figure 6.3.2 . Moving Rigid Wall Definition

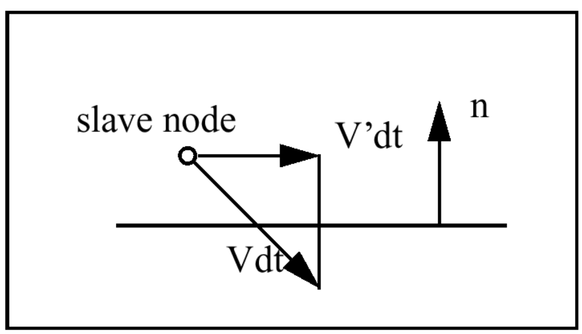


A moving wall is a master slave option. Master node defines the wall position at each time step and imposes velocity on impacted slave nodes. Impacted slave node forces are applied to the master node. The slave node forces are computed with momentum conservation. The mass of the slave nodes is not transmitted to the master node, assuming a large rigid wall mass compared to the impacted slave node mass.

6.3.3 Slave node penetration

Slave node penetration must be checked. Figure 6.3.3 shows how penetration is checked.

Figure 6.3.3 Slave Node Penetration



If penetration occurs, a new velocity must be computed. This new velocity is computed using one of three possible situations. These are:

1. Sliding
2. Sliding with Friction
3. Tied

For a node which is allowed to slide along the face of the rigid wall, the new velocity $\vec{V}'$ is given by:

$$\vec{V}' = \vec{V} - \left(\vec{V} \cdot \vec{n}\right)\vec{n} \qquad \text{EQ. 6.3.3.1}$$

A friction coefficient can be applied between a sliding node and the rigid wall. The friction models are developed in section 8.6.4.

For a node that is defined as tied, once the slave node contacts the rigid wall, its velocity is the same as that of the wall. The node and the wall are tied. Therefore:

$$\vec{V}' = 0 \qquad \text{EQ. 6.3.3.2}$$

6.3.4 Rigid wall impact force

The force exerted by nodes impacting onto a rigid wall is found by calculating the impulse by:

$$\vec{I} = \sum_{i=I}^{N} \vec{F}_i \Delta t = \sum_{i=I}^{N} \Delta m_i \left(\vec{V}_i - \vec{W}\right) \qquad \text{EQ. 6.3.4.1}$$

where

N is the number of penetrated slave nodes

$\vec{W}$ is the wall velocity

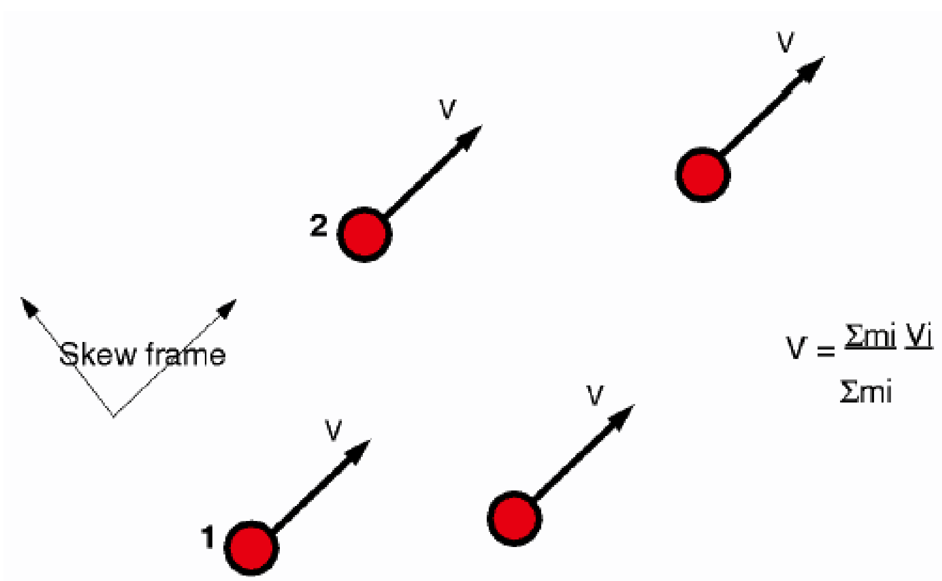
The force can then be calculated by the rate of change in the impulse:

$$\vec{F} = \frac{d\vec{I}}{dt} \qquad \text{EQ. 6.3.4.2}$$

6.4 Rigid Link

A rigid link imposes the same velocity on all slave nodes in one or more directions. The directions are defined to a skew or global frame. Figure 6.4.1 displays a rigid link.

Figure 6.4.1 . Rigid Link Model



The velocity of the group of nodes rigidly linked together is computed using momentum conservation (EQ. 6.5.0.1.). However, no global moment equilibrium is respected.

$$V^i = \frac{\sum_{i=i}^{N} m_i v_i}{\sum_{i=i}^{N} m_i} \qquad \text{EQ. 6.4.0.1}$$

Angular velocity for the i[th] DOF with respect to the global or a skew reference frame is:

$$\omega^i = \frac{\left(\sum_{j=1}^{n} I_j^i \omega_j^i \right)}{\left(\sum_{j=1}^{N} I_j^i \right)} \qquad \text{EQ. 6.4.0.2}$$

For non-coincident nodes, no rigid body motion is possible.

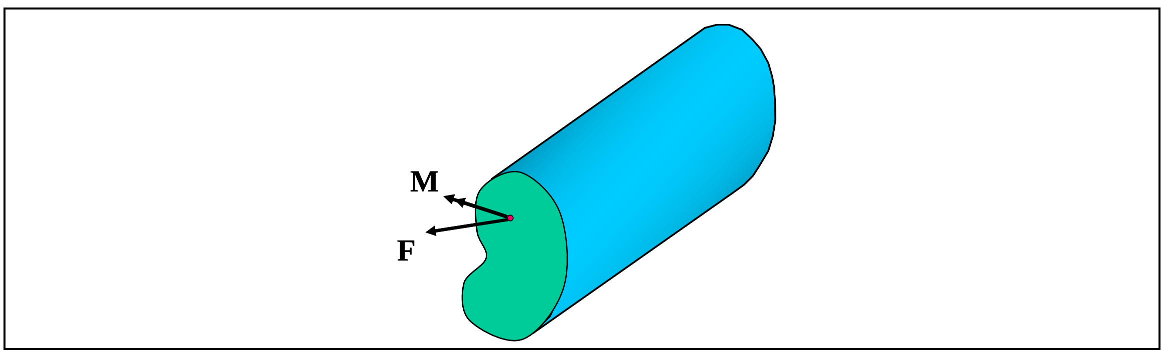
A rigid link is equivalent to an infinitely stiff spring type 8.

6.5 Section

A section is a cut in the structure where forces and moments will be computed and stored in output files. It is defined by:

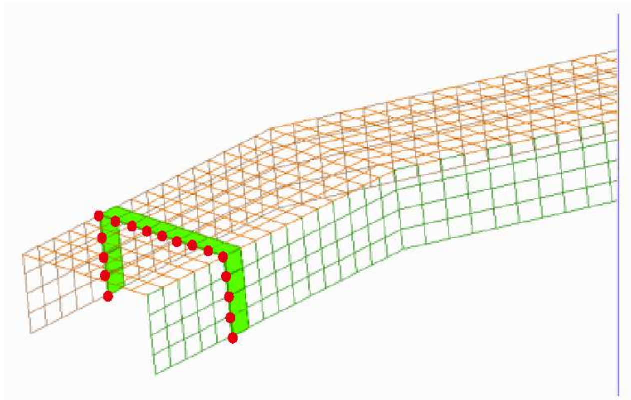
- A cutting plane
- A reference point to compute forces
- A direction of the section.

Figure 6.5.1 Definition of a section for an oriented solid



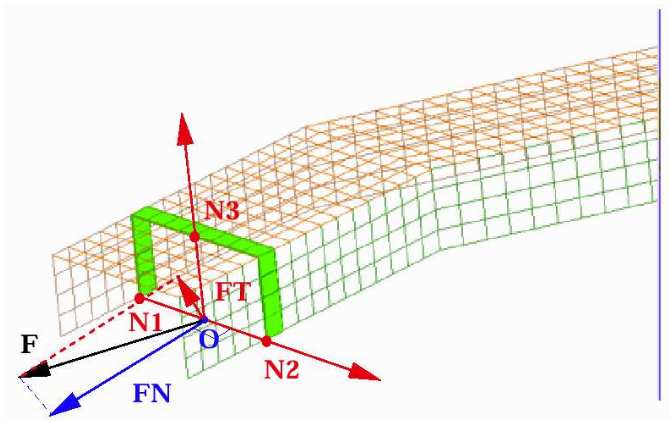
In RADIOSS the cutting plane is defined by a group of elements and its orientation by a group of nodes as shown in Figure 6.5.2.

Figure 6.5.2 Definition of a section for a shell mesh



Then, a point is defined for the center of rotation of the section and a reference frame is attached to this point to compute the internal efforts.

Figure 6.5.3 Center of rotation and its associated frame for a section



The resultant of all forces applied to the elements and its application point are computed by:

$$F = \sum f_i \quad \text{EQ. 6.5.0.1}$$

$$M = \sum m_i + \sum ON_i \times f_i \quad \text{EQ. 6.5.0.2}$$

Figure 6.5.4 Resultant of force and moment for a node I with the rotation point O