

# **RADIOSS THEORY MANUAL**

## **Version 2017 – January 2017**

### **Large Displacement Finite Element Analysis**

#### **Chapter 10**



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# Chapter 10

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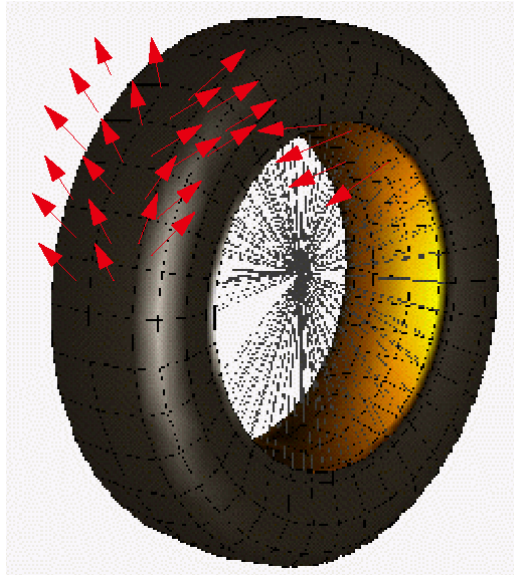
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MONITORED VOLUME

## 10.0 MONITORED VOLUME

An airbag is defined as a monitored volume. A monitored volume is defined as having one or more 3 or 4 node shell property sets. The defined surface must be closed (the normal to shell elements must be all oriented outward as shown in Figure 10.0.1). The shell normal must be oriented outside the volume. It is possible to reverse the shell normals for a given property set (by entering a negative property number). Dummy properties (property type 0) and materials (material type 0) can be used.

**Figure 10.0.1** Tire model: volume closed



There are five types of MONITORED VOLUME:

AREA Type: volume and surface output (post processing option, no pressure)

PRES Type: user function defining pressure versus relative volume

GAS Type: adiabatic pressure volume relation.  $P(V - V_{inc})^\gamma = Cst$  with  $\gamma = \frac{c_p}{c_v}$

AIRBAG Type: Single *airbag*  $PV = nRT = mrT$

$$P = \rho r T = \rho (c_p - c_v) T = (\gamma - 1) \rho c_v T = (\gamma - 1) \frac{E}{V}$$

COMMU Type: Chambered, communicating, folded airbag (*airbag with communications*)

Same basic equations:

Typical use of monitored volume is for tire, fuel tank, airbag.

- For tire use PRES or GAS type monitored volume.
- For fuel tank use PRES or GAS type monitored volume.
- For simple unfolded airbag use AIRBAG type monitored volume.
- For chambered airbag use two or more COMMU type monitored volumes.
- For folded airbag use a set of COMMU type monitored volumes.

### 10.1 AREA type monitored volume

This type is only a post processing option. It allows plotting of the Time History of volume and area of a closed surface.

No pressure is applied with this option.

No specific input is needed for this type.

### 10.2 PRES type monitored volume

With this type the pressure is defined versus relative volume with a tabulated function:

$$P = F(V_0/V)$$

where

F(x): Function

V<sub>0</sub>: Initial volume

No external pressure is defined.

Only the load curve number is needed as specific input.

### 10.3 GAS type monitored volume

This option gives an adiabatic pressure volume relation. With  $\gamma = 1$  an isothermal condition can also be applied.

It is possible to define an incompressible sub-volume to model a volume partially filled with a liquid.

The general equation is:

$$P(V - V_i)^\gamma = P_0(V_0 - V_i)^\gamma$$

With

V<sub>0</sub>: initial volume

V<sub>i</sub>: incompressible volume  $V_i < V_0$

A viscosity  $\mu$  can be used to reduce numerical oscillations.

If  $\mu = 1$  a critical damping (shell mass and volume stiffness) is used. The viscous pressure  $q$  is:

$$q = \mu \frac{1}{A} \sqrt{\frac{\gamma P m_{fabric}}{V}} \frac{dV}{dt}$$

where  $m_{fabric} = A\rho t$  is the mass of fabric and  $A$  its surface.

The applied pressure is  $P - P_{ext} + q$ .

The specific inputs for this type are:

- $\gamma$
- $\mu$
- $P_{ext}$
- $P_0$
- $V_i$

If the deflation is considered (isenthalpic outflow computation), the initial mass of gas, must also be input.

This monitored volume is typically used to model tire pressure or simple fuel tank. For tire model  $V_i$  is zero and for fuel tank  $V_i$  is the fuel volume.

### 10.3.1 Thermodynamical equations

The basic energy equation of the monitored volume can be written as:

$$dE_{\text{Monitored Volume}} = -Pd(V - V_i) - dH_{\text{out}} \quad \text{EQ. 10.3.1.1}$$

where

- E is the internal energy
- P is the pressure
- V is the monitored volume
- $V_i$  incompressible volume
- $H_{\text{out}}$  is the outgoing enthalpy

When the adiabatic condition is applied and assuming a perfect gas:

$$P = \frac{(\gamma - 1)E}{V - V_i} \quad \text{EQ. 10.3.1.2}$$

where  $\gamma$  is the gas constant. For air,  $\gamma = 1.4$ .

The two equations above allow the current volume to be determined. The energy and pressure can then be found.

### 10.3.2 Variation of external work

At the current time step, t, assume we know:

- $P(t-dt)$
- $E(t-dt)$
- $\tilde{V} = V - V_i$

$\delta W(t)$ ,  $E(t)$ ,  $P(t)$  will be obtained as follows. Let:

$$\delta W = \frac{P(t) + P(t - dt)}{2} (\tilde{V}(t) - \tilde{V}(t - dt)) \quad \text{EQ. 10.3.2.1}$$

be the variation of external work and from the adiabatic condition:

$$P = \frac{(\gamma - 1)E}{\tilde{V}} \quad \text{EQ. 10.3.2.2}$$

we have:

$$\delta W = \frac{(\gamma - 1)\Delta\tilde{V}}{2} \left[ \frac{E(t)}{\tilde{V}(t)} + \frac{E(t - dt)}{\tilde{V}(t - dt)} \right] \quad \text{EQ. 10.3.2.3}$$

Let now:

$$E = E(t-dt)$$

$$\tilde{V} = (\tilde{V}(t) - \tilde{V}(t-dt)) / 2$$

$$\Delta E = E(t) - E(t-dt)$$

$$\delta W = \frac{(\gamma-1) \Delta \tilde{V}}{2} \frac{\Delta \tilde{V}}{\tilde{V}} \left[ \frac{1 + \frac{\Delta E}{E}}{1 + \frac{\Delta \tilde{V}}{2\tilde{V}}} + \frac{1}{1 - \frac{\Delta \tilde{V}}{2\tilde{V}}} \right] \quad \text{EQ. 10.3.2.4}$$

Hence, the external work is given by:

$$\delta W \approx (\gamma-1) E \frac{\Delta \tilde{V}}{\tilde{V}} \left[ 1 + \frac{\Delta E}{2E} \right] \quad \text{EQ. 10.3.2.5}$$

Computing the energy from basic principles:

$$\Delta \tilde{E} = -(\gamma-1) E \frac{\Delta \tilde{V}}{\tilde{V}} \left[ 1 + \frac{\Delta E}{2E} \right] - \Delta H_{out} \quad \text{EQ. 10.3.2.6}$$

$\Delta H_{out}$  can be estimated from,  $u(t-dt)$ , the velocity at vent hole; this estimation will be described hereafter.

The variation of internal energy  $\Delta E$  can be given by:

$$\Delta E = \left[ -(\gamma-1) E \frac{\Delta \tilde{V}}{\tilde{V}} - \Delta H_{out} \right] \left[ 1 - (\gamma-1) E \frac{\Delta \tilde{V}}{2\tilde{V}} \right] \quad \text{EQ. 10.3.2.7}$$

Therefore:

$$E(t) = E(t-dt) + \Delta E \quad \text{EQ. 10.3.2.8}$$

$$P(t) = (\gamma-1) \frac{E(t)}{\tilde{V}(t)} \quad \text{EQ. 10.3.2.9}$$

This pressure is then applied to the monitored volume to get:

1. New accelerations
2. New velocities
3. New geometry
4. New volume
5. Ready for next step evaluation

### 10.3.3 Venting

Venting, or the expulsion of gas from the volume, is assumed to be *isenthalpic*.

The flow is also assumed to be unshocked, coming from a large reservoir and through a small orifice with effective surface area,  $A$ .

Conservation of enthalpy leads to velocity,  $u$ , at the vent hole. The Bernouilli equation is then written as:

$$\text{(monitored volume)} \frac{\gamma}{\gamma-1} \frac{P}{\rho} = \frac{\gamma}{\gamma-1} \frac{P_{ext}}{\rho_{vent}} + \frac{u^2}{2} \text{(vent hole)} \quad \text{EQ. 10.3.3.1}$$

Applying the adiabatic conditions:

$$(\text{monitored volume}) \frac{P}{\rho^\gamma} = \frac{P_{ext}}{\rho_{vent}^\gamma} \quad (\text{vent hole}) \quad \text{EQ. 10.3.3.2}$$

Therefore, the exit velocity is given by:

$$u^2 = \frac{2\gamma}{\gamma-1} \frac{P}{\rho} \left( 1 - \left( \frac{P_{ext}}{P} \right)^{\frac{\gamma-1}{\gamma}} \right) \quad \text{EQ. 10.3.3.3}$$

The mass flow rate is given by:

$$\dot{m}_{out} = \rho_{vent} A_{vent} u = \rho \left( \frac{P_{ext}}{P} \right)^{1/\gamma} A_{vent} u \quad \text{EQ. 10.3.3.4}$$

The energy flow rate is given by:

$$\dot{E}_{out} = \dot{m} \frac{E}{\rho \tilde{V}} = \left( \frac{P_{ext}}{P} \right)^{1/\gamma} A_{vent} u \frac{E}{\tilde{V}} \quad \text{EQ. 10.3.3.5}$$

The vent hole area or scale factor area,  $A_{vent}$ , can be defined in two ways:

- a constant area taking into account a discharge coefficient
- a variable area equal to the area of a specified surface, multiplied by a discharge coefficient.
- a variable area equal to the area of the deleted elements within a specified surface, multiplied by a discharge coefficient.

### 10.3.4 Supersonic outlet flow

Vent pressure  $P_{vent}$  is equal to external pressure  $P_{ext}$  for unshocked flow. For shocked flow,  $P_{vent}$  is equal to critical pressure  $P_{crit}$  and  $U$  is bounded to critical sound speed:

$$u^2 < \frac{2}{\gamma+1} c^2 = \frac{2}{\gamma+1} \frac{P}{\rho} \quad \text{EQ. 10.3.4.1}$$

And,

$$P_{crit} = P \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{vent} = \max(P_{crit}, P_{ext})$$

### 10.3.5 GAS type monitored volume examples

Some applications in RADIOSS:

- A tire model :  
The inputs are:
  - $\gamma = 1.4$
  - $\mu = 0$ .
  - $P_{ext} = 10^5 \text{ Pa}$
  - $P_{ini}$  = initial tire pressure
 Then, the pressure in the tire is  $P_{tire} = P_{ext} - P_{ini}$ 
  - $V_{inc} = 0$



- A fuel tank model if the sloshing effect is neglected  
 Only if the sloshing effect is neglected, pressure in a partial filled fuel tank can be modeled with a type GAS monitored volume. Use following input:
  - $\gamma = 1.4$
  - $\mu = 0.01$
  - $P_{ext} = 10^5 \text{ Pa}$
  - $P_{ini} = 10^5 \text{ Pa}$
  - $V_{inc} = \text{volume of fuel}$

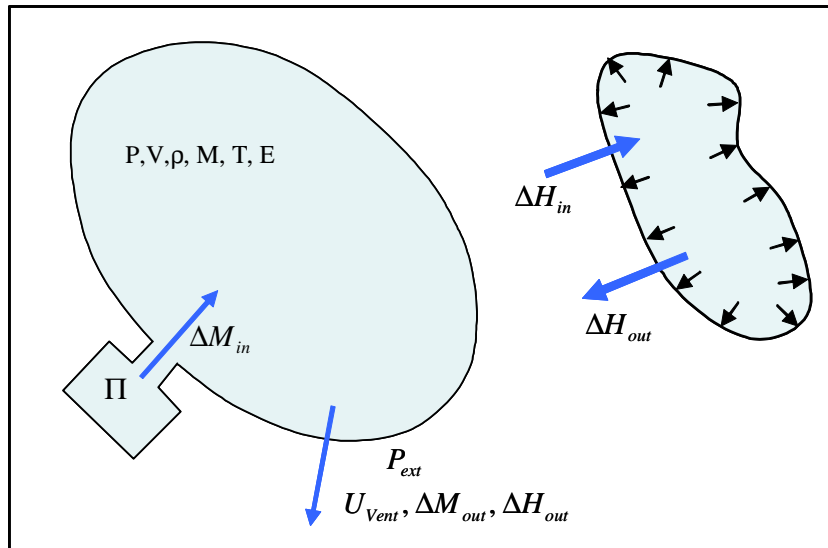
### 10.4 AIRBAG type monitored volume

The airbag simulation used by RADIOSS uses a special uniform pressure airbag. Hence, regardless of state of inflation or shape, the pressure remains uniform.

Perfect gas law and adiabatic conditions are assumed. Injected mass and temperature are defined as a time function. A sensor can define the inflator starting time.

Deflation of vent hole is available after reaching a pressure ( $P_{def}$ ) for a given duration  $\Delta t P_{def}$  or at time ( $T_{def}$ ) criteria.  $\Delta t P_{def}$  prevents deflation due to initial peak of pressure corresponding to the jetting activation.

Figure 10.4.1 Uniform Pressure Airbag



The key assumptions are:

- Uniform airbag pressure <sup>TM</sup> kinetic energy is negligible
- Adiabatic conditions

The airbag simulation must include:

- Injection of energy and mass
- Bag mechanics (i.e. unfolding, expansion, membrane tension, impacts ...)
- Exhaust through vent holes

### 10.4.1 Thermodynamical equations

The basic energy equation of the airbag can be written as:

$$dE_{airbag} = -PdV + dH_{in} - dH_{out} \quad \text{EQ. 10.4.1.1}$$

Where:

$E$  is the internal energy

$P$  is the pressure

$V$  is the airbag volume

$H_{in}$  is the incoming enthalpy

$H_{out}$  is the outgoing enthalpy

When the adiabatic condition is applied and assuming a perfect gas:

$$P = \frac{(\gamma - 1)E}{V} \quad \text{EQ. 10.4.1.2}$$

where  $\gamma$  is the gas constant. For air,  $\gamma = 1.4$

The two equations above allow the current airbag volume to be determined. The energy and pressure can then be found. To know the current airbag volume, derive energy and thus pressure.

Considering a gas such that the constant pressure and the constant volume heat capacities per mass unit (respectively,  $c_p$  and  $c_v$ ) vary in temperature  $T$ .

The following temperature dependency of the constant pressure heat capacity is assumed:

$$c_p = a + bT + cT^2 \quad \text{EQ. 10.4.1.3}$$

where  $a$ ,  $b$  and  $c$  are the constants depending to characteristics of the gas.

The  $c_p$  and  $c_v$  satisfy the Mayer relation:

$$c_p(T) - c_v(T) = \frac{R}{M} \quad \text{EQ. 10.4.1.4}$$

With  $R$  is the universal gas constant depending to the unit system ( $R=8.3144 \text{ Jmol}^{-1}\text{K}^{-1}$ ):

$$c_v(T) = \frac{de}{dT}(T) \quad h(t) = e(t) + \frac{P}{\rho} \quad \text{EQ. 10.4.1.5}$$

Where,  $e$  is the specific energy and  $h$  the specific enthalpy per mass unit of the gas at temperature  $T$ . We can then obtain:

$$e(T) = e_{cold} + \int_{T_{cold}}^T c_v(T)dT \quad \text{and} \quad h(T) = e_{cold} + \int_{T_{cold}}^T c_v(T)dT + \frac{R}{M}T \quad \text{EQ. 10.4.1.6}$$

Where, the lower index *cold* refers to the reference temperature  $T_{cold}$ .

Now, assuming an ideal mixture of gas:

$$PV = nRT \quad \text{EQ. 10.4.1.7}$$

with  $n$  the total number of moles:

$$n = \sum_i n^{(i)} = \sum_i \frac{m^{(i)}}{M^{(i)}} \quad \text{EQ. 10.4.1.8}$$

where  $m^{(i)}$  is the mass of gas  $i$ , and  $M^{(i)}$  is the molar weight of gas  $i$ .

It follows:

$$P = \frac{nRT}{V} \quad \text{EQ. 10.4.1.9}$$

With  $n = \sum_i \frac{m^{(i)}}{M^{(i)}}$ .

### 10.4.2 Energy variation within a time step

Let  $T(t-\delta t)$  the temperature,  $P(t-\delta t)$  the pressure, and  $V(t-\delta t)$  the volume of the airbag at time  $t-\delta t$ , and  $m^{(i)}$  the mass of gas  $i$  at time  $t-\delta t$ .  $T(t)$ ,  $P(t)$ ,  $V(t)$  are respectively temperature, pressure and volume of the airbag at time  $t$ , and  $m^{(i)} + \delta m_{in}^{(i)} - \delta m_{out}^{(i)}$  the mass of gas  $i$  at time  $t$ .

Using EQ. 10.4.1.5, the variation of total gas energy can be written as:

$$\Delta E = \left[ \sum_i (m^{(i)} + \delta m_{in}^{(i)} - \delta m_{out}^{(i)}) \left( e^{(i)_{cold}} + \int_{T_{cold}}^{T(t)} c_v^{(i)}(T) dT \right) \right] - \left[ \sum_i m^{(i)} \left( e^{(i)_{cold}} + \int_{T_{cold}}^{T(t-\delta)} c_v^{(i)}(T) dT \right) \right]$$

EQ. 10.4.2.1

which can be written as:

$$\Delta E = \left[ \sum_i (m^{(i)} + \delta m_{in}^{(i)} - \delta m_{out}^{(i)}) \left( \int_{T(t-\delta)}^{T(t)} c_v^{(i)}(T) dT \right) \right] + \left[ \sum_i (\delta m_{in}^{(i)} - \delta m_{out}^{(i)}) \left( e^{(i)_{cold}} + \int_{T_{cold}}^{T(t-\delta)} c_v^{(i)}(T) dT \right) \right]$$

EQ. 10.4.2.2

On the other hand, the basic energy equation EQ. 10.4.1.1 of the airbag and the expression of enthalpy in EQ. 10.4.1.5 gives:

$$\Delta E = \left[ \sum_i \delta m_{in}^{(i)} \left( e^{(i)_{cold}} + \frac{R}{M^{(i)}} T^{(i)_{in}} + \int_{T_{cold}}^{T^{(i)_{in}}} c_v^{(i)}(T) dT \right) \right] - \left[ \sum_i \delta m_{out}^{(i)} \left( e^{(i)_{cold}} + \frac{R}{M^{(i)}} T^{(i)_{out}} + \int_{T_{cold}}^{T^{(i)_{out}}} c_v^{(i)}(T) dT \right) \right] - \delta W$$

EQ. 10.4.2.2

Where  $\delta m_{in}^{(i)}$  and  $T^{(i)_{in}}$  are characteristics of the inflator and are considered as input to the problem.  $\delta m_{out}^{(i)}$  and  $T^{(i)_{out}}$  can be estimated from the velocity at vent hole  $u(t)$ .  $\delta W$  is the variation of the external work. This estimation will be described hereafter.

It comes from EQ. 10.4.2.1 and EQ. 10.4.2.2:

$$\sum_i \left( m^{(i)} + \delta m^{(i)}_{in} - \delta m^{(i)}_{out} \right) \int_{T(t-\delta t)}^{T(t)} c_v^{(i)}(T) dT = \left[ \sum_i \delta m^{(i)}_{in} \left( \frac{R}{M^{(i)}} T^{(i)}_{in} + \int_{T(t-\delta t)}^{T^{(i)}_{in}} c_v^{(i)}(T) dT \right) \right] - \left[ \sum_i \delta m^{(i)}_{out} \left( \frac{R}{M^{(i)}} T^{(i)}_{out} + \int_{T(t-\delta t)}^{T^{(i)}_{out}} c_v^{(i)}(T) dT \right) \right] - \delta W \quad \text{EQ. 10.4.2.3}$$

The variation of the external work can be written as:

$$\delta W = \frac{(P(t) + P(t - \delta t))}{2} (V(t) - V(t - \delta t)) \quad \text{EQ. 10.4.2.4}$$

Using EQ. 10.4.1.8, the last expression can be written as:

$$\delta W = \frac{1}{2} \left( \frac{\left[ \sum_i \frac{m^{(i)} + \delta m^{(i)}_{in} - \delta m^{(i)}_{out}}{M^{(i)}} \right] RT(t)}{V(t)} + \frac{\left[ \sum_i \frac{m^{(i)}}{M^{(i)}} \right] RT(t - \delta t)}{V(t - \delta t)} \right) (V(t) - V(t - \delta t)) \quad \text{EQ. 10.4.2.5}$$

The last equation can be introduced to EQ. 10.4.2.3:

$$\begin{aligned} & \left[ \sum_i \left( m^{(i)} + \delta m^{(i)}_{in} - \delta m^{(i)}_{out} \right) \int_{T(t-\delta t)}^{T(t)} c_v^{(i)}(T) dT \right] + \left[ \sum_i \frac{m^{(i)} + \delta m^{(i)}_{in} - \delta m^{(i)}_{out}}{M^{(i)}} \right] RT(t) \frac{V(t) - V(t - \delta t)}{2V(t)} \\ &= \left[ \sum_i \delta m^{(i)}_{in} \left( \frac{R}{M^{(i)}} T^{(i)}_{in} + \int_{T(t-\delta t)}^{T^{(i)}_{in}} c_v^{(i)}(T) dT \right) \right] - \left[ \sum_i \delta m^{(i)}_{out} \left( \frac{R}{M^{(i)}} T^{(i)}_{out} + \int_{T(t-\delta t)}^{T^{(i)}_{out}} c_v^{(i)}(T) dT \right) \right] - \\ & \left[ \sum_i \frac{m^{(i)}}{M^{(i)}} \right] RT(t - \delta t) \frac{V(t) - V(t - \delta t)}{2V(t - \delta t)} \end{aligned} \quad \text{EQ. 10.4.2.6}$$

The first order approximation  $\int_{T(t-\delta t)}^{T(t)} c_v^{(i)}(T) \approx c_v^{(i)}(T_{|t-\delta t})(T(t) - T(t - \delta t))$  for each gas, which allows rewrite EQ. 10.4.2.6 as:

$$\begin{aligned} & \left[ \sum_i \left( m^{(i)} + \delta m^{(i)}_{in} - \delta m^{(i)}_{out} \right) c_v^{(i)}(T_{|t-\delta t})(T(t) - T(t - \delta t)) \right] + \\ & \left[ \sum_i \frac{m^{(i)} + \delta m^{(i)}_{in} - \delta m^{(i)}_{out}}{M^{(i)}} \right] RT(t) \frac{V(t) - V(t - \delta t)}{2V(t)} \\ &= \left[ \sum_i \delta m^{(i)}_{in} \left( \frac{R}{M^{(i)}} T^{(i)}_{in} + \int_{T(t-\delta t)}^{T^{(i)}_{in}} c_v^{(i)}(T) dT \right) \right] - \left[ \sum_i \delta m^{(i)}_{out} \left( \frac{R}{M^{(i)}} T^{(i)}_{out} + \int_{T(t-\delta t)}^{T^{(i)}_{out}} c_v^{(i)}(T) dT \right) \right] - \\ & \left[ \sum_i \frac{m^{(i)}}{M^{(i)}} \right] RT(t - \delta t) \frac{V(t) - V(t - \delta t)}{2V(t - \delta t)} \end{aligned} \quad \text{EQ. 10.4.2.7}$$

Which allows to determine the actual temperature  $T(t)$ . The actual pressure then computed from the equation of perfect gas (EQ. 10.4.1.8).

### 10.4.3 Mass injection

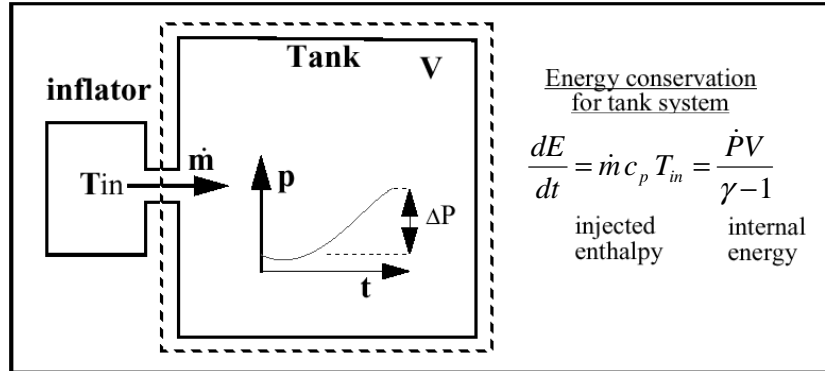
The amount of mass injected into the airbag needs to be defined with respect to time. This is required as a function. The specific heat,  $c_p$ , along with a function defining the change in temperature with time is required.

The data can be obtained by two methods:

1. Possibly from the airbag manufacturer
2. From a tank experiment

A diagram of a tank experiment can be seen in Figure 10.4.2.

Figure 10.4.2 Tank Experiment



The mass versus time curve can be derived from the pressure curve if  $T_{in}$  is known:

$$\dot{m} = \frac{\dot{P}VM}{\gamma RT_{in}} \tag{EQ. 10.4.3.1}$$

Where,  $M$  is the molecular weight of the injected gas.  $R$  is the *perfect gas constant* :  $\frac{R}{M} = c_p - c_v$ ;  $\gamma = \frac{c_p}{c_v}$ .

The average estimate for temperature of injection is:

$$T_{in} = \frac{\Delta P VM}{\Delta m \gamma R} \tag{EQ. 10.4.3.2}$$

where:

$\Delta P$  is the total pressure variation during the experiment.

$\Delta m$  is the total injected mass, which can be derived from the mass of propellant in the pyrotechnic inflator and the chemical reaction; 40% is a typical value for the ratio of the produced mass of gas to the solid propellant mass.

### 10.4.4 Venting: outgoing mass determination

Venting, or the expulsion of gas from the airbag, is assumed to be *isenthalpic*.

The flow is also assumed to be unshocked, coming from a large reservoir and through a small orifice with effective surface area,  $A$ .

Conservation of enthalpy leads to velocity,  $u$ , at the vent hole. The Bernoulli equation is then written as:

$$\text{(airbag)} \quad \frac{\gamma}{\gamma-1} \frac{P}{\rho} = \frac{\gamma}{\gamma-1} \frac{P_{ext}}{\rho_{vent}} + \frac{u^2}{2} \quad \text{(vent hole)} \quad \text{EQ. 10.4.4.1}$$

Applying the adiabatic conditions:

$$\text{(airbag)} \quad \frac{P}{\rho^\gamma} = \frac{P_{ext}}{\rho_{vent}^\gamma} \quad \text{(vent hole)} \quad \text{EQ. 10.4.4.2}$$

Therefore, the exit velocity is given by:

$$u^2 = \frac{2\gamma}{\gamma-1} \frac{P}{\rho} \left( 1 - \left( \frac{P_{ext}}{P} \right)^{\frac{\gamma-1}{\gamma}} \right) \quad \text{EQ. 10.4.4.3}$$

with  $\rho = \frac{\sum_i m^{(i)}}{V}$  the averaged density of the gas and  $\gamma = \frac{\left[ \frac{\sum_i m^{(i)} c_p^{(i)}}{\sum_i m^{(i)}} \right] / \left[ \frac{\sum_i m^{(i)} c_v^{(i)}}{\sum_i m^{(i)}} \right]}{\left[ \frac{\sum_i m^{(i)} c_p^{(i)}}{\sum_i m^{(i)}} \right] / \left[ \frac{\sum_i m^{(i)} c_v^{(i)}}{\sum_i m^{(i)}} \right]}$  the fraction of massic

averages of heat capacities at constant pressure and constant volume.

The mass flow rate is given by:

$$\dot{m}_{out} = \rho_{vent} A_{vent} u = \rho \left( \frac{P_{ext}}{P} \right)^{1/\gamma} A_{vent} u \quad \text{EQ. 10.4.4.4}$$

The energy flow rate is given by:

$$\dot{E}_{out} = \dot{m} \frac{E}{\rho V} = \left( \frac{P_{ext}}{P} \right)^{1/\gamma} A_{vent} u \frac{E}{V} \quad \text{EQ. 10.4.4.5}$$

The total mass flow rate is given by  $dm_{out} = \rho \left( \frac{P_{ext}}{P} \right)^{1/\gamma} A_{vent} u \quad \text{EQ. 10.4.4.6}$

where  $A_{vent}$  is the vent hole surface.

The vent hole area or scale factor area,  $A_{vents}$ , can be defined in two ways:

- a constant area taking into account a discharge coefficient
- a variable area equal to the area of a specified surface multiplied by a discharge coefficient.

**10.4.4.1 Supersonic outlet flow**

Vent pressure  $P_{vent}$  is equal to external pressure  $P_{ext}$  for unshocked flow. For shocked flow,  $P_{vent}$  is equal to critical pressure  $P_{crit}$  and  $u$  is bounded to critical sound speed:

$$u^2 < \frac{2}{\gamma+1} c^2 = \frac{2^* \gamma}{\gamma^* + 1} \frac{P}{\rho} \quad \text{EQ. 10.4.4.7}$$

And,

$$P_{crit} = P \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{vent} = \max(P_{crit}, P_{ext})$$

**10.4.4.2 Outgoing mass per gas**

The mass flow of gas  $i$  is  $dm^{(i)}_{out} = \frac{V^{(i)}}{V} dm_{out}$ , where  $V^{(i)}$  is the volume occupied by gas  $i$  and satisfies:

$$V^{(i)} = \frac{n^{(i)}}{n} V \text{ (from } PV^{(i)} = n^{(i)}RT \text{ and } PV = \left[ \sum_i n^{(i)} \right] RT \text{ )}.$$

It comes finally

$$dm^{(i)}_{out} = \frac{n^{(i)}}{\sum_i n^{(i)}} dm_{out} \tag{EQ. 10.4.4.8}$$

**10.4.5 Porosity**

The isenthalpic model is also used for porosity. In this case, one can define the outgoing surface by the following equation:

$$A_{OUT} = \lambda f(A) g(P) h(t) \tag{EQ. 10.4.5.1}$$

where:

- $A$  : Area of the specified surface.
- $\lambda$  : Scale factor
- $P$  : Pressure of the gas
- $t$  : Time

It is also possible to define closure of the porous surface in the case of contact.

The second model integrated in RADIOSS is called Chemkin model in which the mass flow due to the porosity is computed by:

$$\dot{m} = A_{vent} u \rho_{airbag} \tag{EQ. 10.4.5.2}$$

where  $u$  is user defined function represented as :

$$u = f(P - P_{ext}) \tag{EQ. 10.4.5.3}$$

**10.4.6 Initial conditions**

To avoid initial disequilibrium and mathematical discontinuity for zero mass or zero volume, following initial conditions are set:

- $P_{ini} = P_{ext}$
- $T_{ini}$

- If the initial volume is less than  $10^{-4} A^{\frac{3}{2}}$  a constant small volume is added to obtain an initial volume:

$$V_{ini} = 10^{-4} A^{\frac{3}{2}}$$

- Initial mass energy density is defined from the above values.

There is no need to define an injected mass at time zero.

### 10.4.7 Jetting effect

The jetting effect is modeled as an overpressure applied to each element of the airbag (Figure 10.4.3).

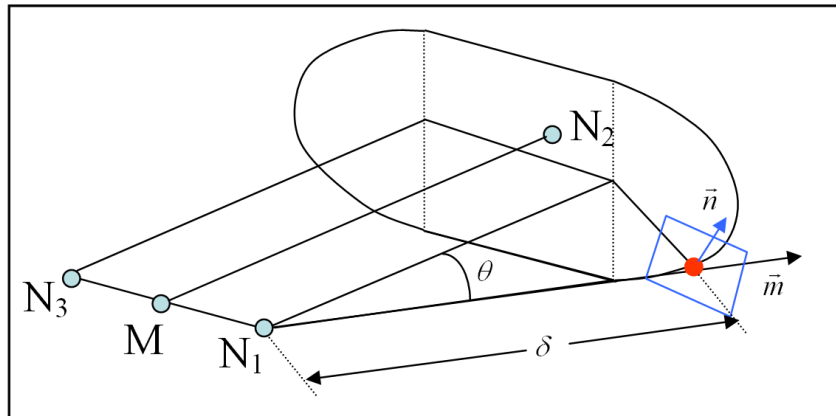
$$\Delta P_{jet} = \Delta P_1(t) \cdot \Delta P_2(\theta) \cdot \Delta P_3(\delta) \cdot \max(\vec{n} \cdot \vec{n}_1, 0) \tag{EQ. 10.4.7.1}$$

with:

- $\vec{n}_1$  being the normalized vector between the projection of the center of the element upon segment (N1,N3) and the center of element as shown in the figure.
- $\theta$  is the angle between the vector  $\overrightarrow{MN_2}$  and the vector  $\vec{n}_1$ .
- $\delta$  the distance between the center of the element and its projection of a point upon segment (N1, N3).

The projection upon the segment (N1, N3) is defined as the projection of the point in direction  $\overrightarrow{MN_2}$  upon the line (N1, N3) if it lies inside the segment (N1, N3). If this is not the case, the projection of the point upon segment (N1, N3) is defined as the closest node N1 or N3. If N3 coincides to N1, the dihedral shape of the jet is reduced to a conical shape.

Figure 10.4.3 Jetting effect schema



### 10.4.8 Reference Metric

This option can be used to inflate an airbag instead of simulating the real unfolding which is difficult numerically. A jetting effect can be added in order to set a preferential direction for the unfolding.

An initial State (airbag geometry input by the user in the initial deck D00) is given.

Reference Metric is a reference state with no strains. This state is defined from a file containing the nodal coordinates, the connectivities of the airbag being the same as in the Initial State. The format of this file is the same as the format of Nodal Coordinates of a RADIOSS Starter input. The file is read according to /XREF keyword of a RADIOSS Starter input deck. The default name is RunNameRS0.

The Reference Metric method consists in the calculation for the Initial State of the initial strains, stresses and energy.

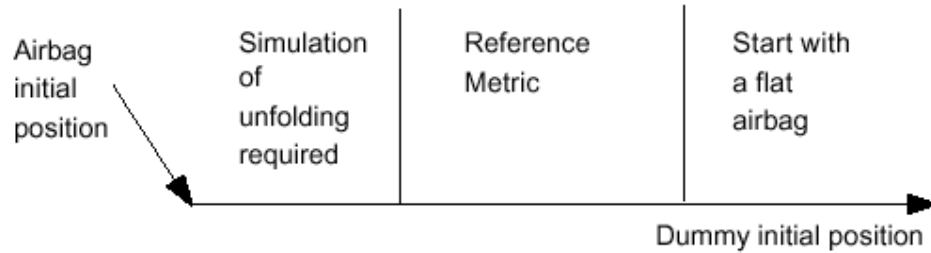
Compatible with:

- 4-node shells
- 3-node shells



- large strain option
- small strain option
- material law # 1
- material law # 19

Figure 10.4.4 Reference metric schema



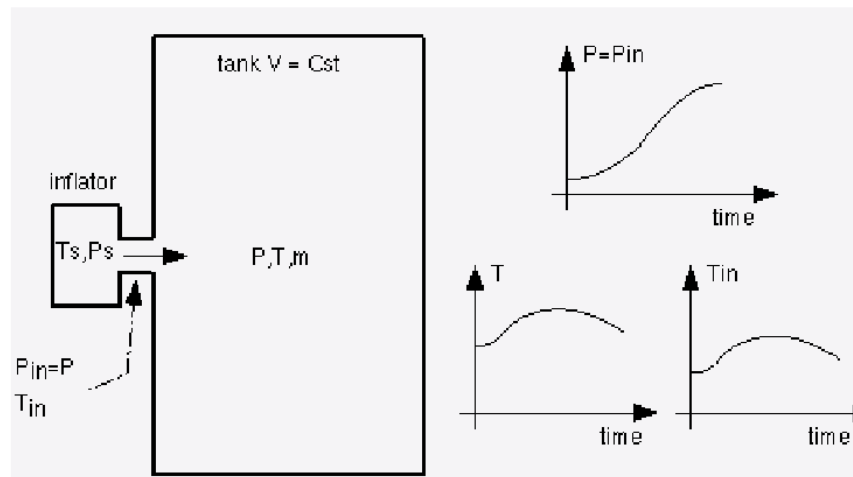
### 10.4.9 Tank experiment

Injected mass curve and injection temperature can be obtained from:

- the airbag manufacturer
- a tank test

With a tank test it is possible to measure temperature at injection point or in the middle of the tank. For pressure the two values are equal.

Figure 10.4.5 Tank experiment schema



#### 10.4.9.1 $T_{in}$ and $P$ are known:

$$\dot{m} = \frac{\frac{dP}{dt} V}{\gamma(\gamma-1)C_v T_{in}}$$

EQ. 10.4.9.1

**10.4.9.2 T and P are known:**

$$\dot{m} = \frac{\left( T \frac{dP}{dt} - P \frac{dT}{dt} \right) V}{\gamma(\gamma-1)C_v T^2} \quad \text{EQ. 10.4.9.2}$$

Or,

$$m = \frac{PV}{(\gamma-1)C_v T} \quad \text{EQ. 10.4.9.3}$$

$$T_{in} = \frac{T^2 \frac{dP}{dt}}{\gamma \left( T \frac{dP}{dt} - P \frac{dT}{dt} \right)} \quad \text{EQ. 10.4.9.4}$$

Or,

$$T_{in} = \frac{T}{\gamma} + \frac{m}{\gamma \dot{m}} \frac{dT}{dt} \quad \text{EQ. 10.4.9.5}$$

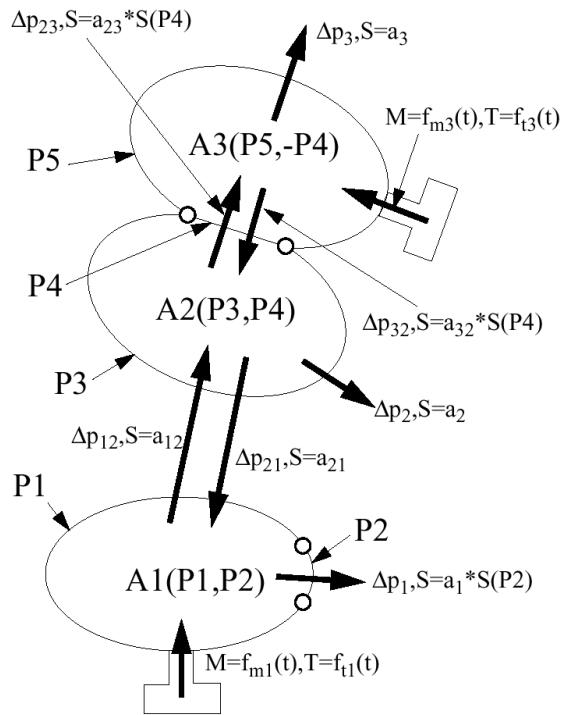
And if  $T$  is constant:  $T_{in} = T / \gamma$ **10.5 Monitored volume type COMMU**

The airbag simulation used by RADIOSS adopts a special uniform pressure airbag. Hence, regardless of the state of inflation or shape, the pressure is uniform.

Perfect gas law and adiabatic conditions are assumed. Injected mass and temperature are defined as a time function. A sensor can define the inflate start time.

Deflation of vent hole is available after reaching a pressure (Pdef) or time (Tdef) criteria.

Figure 10.5.1 Chambered airbag schema



The key assumptions are:

- Uniform airbag pressure  $\Leftrightarrow$  kinetic energy is negligible
- Adiabatic conditions

The airbag simulation must include:

- Injection of energy and mass
- Bag mechanics (i.e. unfolding, expansion, membrane tension, impacts, ...)
- Exhaust through vent holes

This option is used to simulate chambered airbags and may be used to unfold an airbag.

Each COMMU type monitored volume works like an AIRBAG type monitored volume with possible vent communication with some other monitored volume of COMMU type. A chambered airbag is therefore modeled with two or more COMMU type monitored volumes.

Each monitored volume can have an inflator and an atmospheric vent hole.

Monitored volume 1 can communicate with monitored volume 2 with or without communication from 2 to 1. Communicating area, deflation pressure or time from 1 to 2 can be different from corresponding values from 2 to 1. It is thereby possible to model a valve communication.

### 10.5.1 Thermodynamical equations

Same equations as for AIRBAG type monitored volume are used. Refer to Chapter 10.4.1.

### 10.5.2 Variation of external work

Same equations as for AIRBAG type monitored volume are used. Refer to Chapter 10.4.2.

### 10.5.3 Mass injection

Same equations as for AIRBAG type monitored volume are used. Refer to Chapter 10.4.3.

### 10.5.4 Venting

Same equations as for AIRBAG type monitored volume are used. Refer to Chapter 10.4.4.

The mass flow rate is given by:

$$\dot{m}_{out} = \rho_{vent} A_{vent} u = \rho \left( \frac{P_{ext}}{P} \right)^{1/\gamma} A_{vent} u \quad \text{EQ. 10.5.4.1}$$

The energy flow rate is given by:

$$\dot{E}_{out} = \dot{m} \frac{E}{\rho V} = \left( \frac{P_{ext}}{P} \right)^{1/\gamma} A_{vent} u \frac{E}{V} \quad \text{EQ. 10.5.4.2}$$

These mass and energy flux are removed from the current volume and added to the communicating volume at next cycle.

### 10.5.5 Supersonic outlet flow

Same equations as for AIRBAG type monitored volume is used. Refer to Chapter 10.4.4.1.

### 10.5.6 Jetting effect

Same explanation as for AIRBAG type monitored volume is used. Refer to Chapter 10.4.7.

### 10.5.7 Reference Metric

Same explanation as for AIRBAG type monitored volume is used. Refer to Chapter 10.4.8.

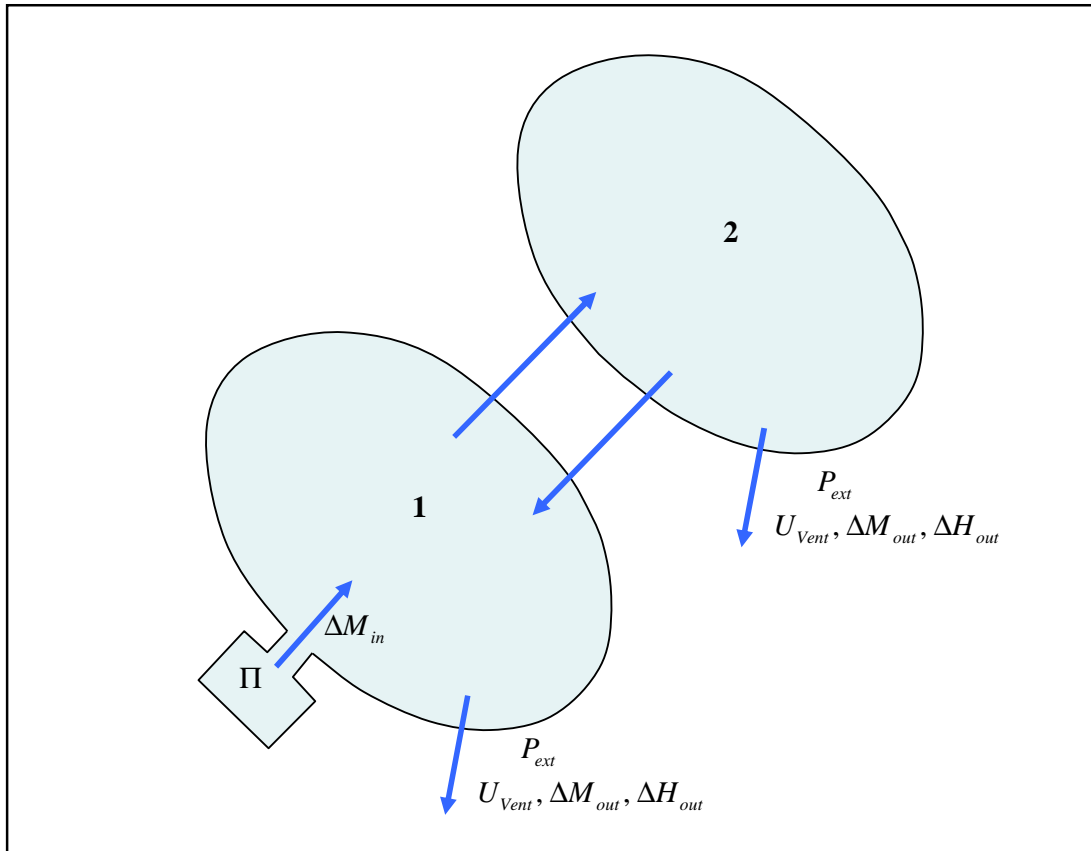
## 10.5.8 Examples of COMMU type monitored volume

### 10.5.8.1 Example 1

Volume 1 communicates with volume 2 and vice-versa.

Monitored volume 1 communicates with monitored volume 2 with or without communication from 2 to 1. The communicating area, deflation pressure or time from 1 to 2 can be different from the corresponding values from 2 to 1. It is thereby possible to model a valve communication.

Figure 10.5.2 Example 1: communication between the 2 volumes



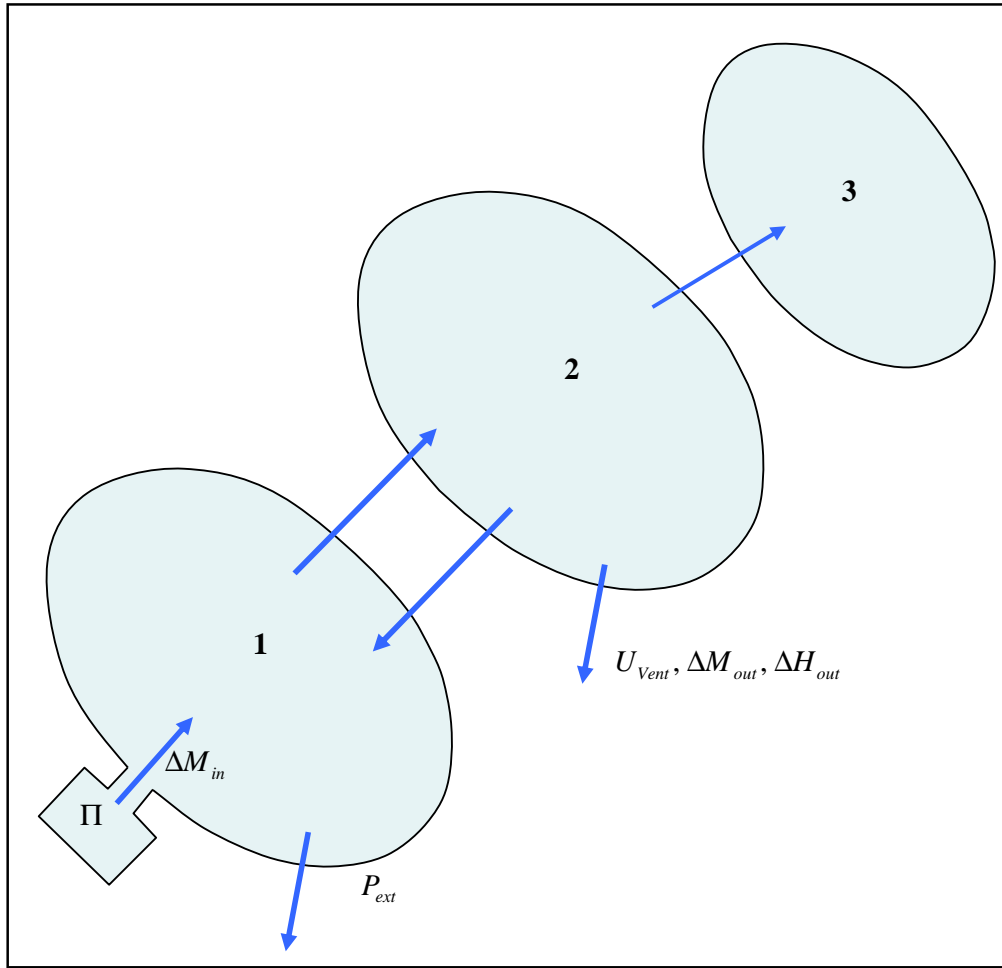
### **10.5.8.2 Example 2**

Volume 1 communicates with volume 2 and volume 2 with volumes 1 and 3, but there is no communication from 3 to 2.

Two COMMU type monitored volume communications can have common nodes or common shell property sets but this is optional.

To model a folded airbag, one COMMU type monitored volume is used for each folded part. The boundary between two folded parts is closed with a dummy property set (fictitious property). The pressure in each folded part will be different and the area of communication will increase during inflation. With this model, the volume with inflator will inflate first and before than folded parts (better than "jetting" model).

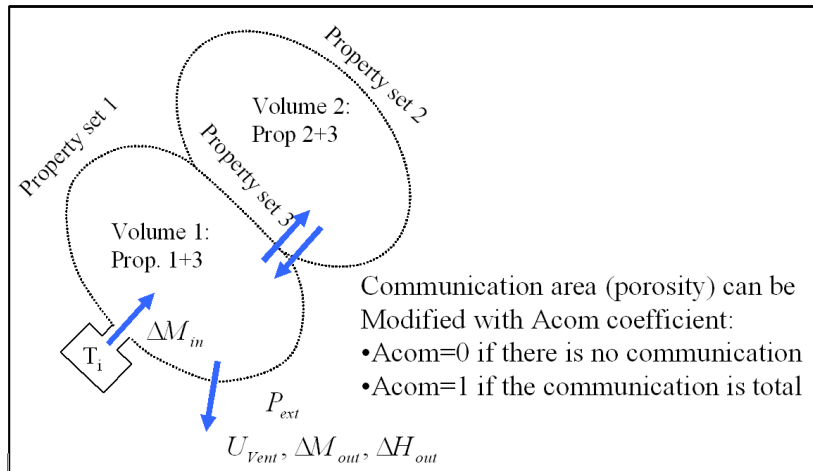
Figure 10.5.3 Example 2: no communication between 2 and 3



**10.5.8.3 Example 3**

Volume 1 and volume 2 with common property set.

Figure 10.5.4 Monitored volume with communication coefficient



**10.5.8.4 Example 4: a folded airbag**

