



# **RADIOSS THEORY MANUAL**

## **Version 2017 – January 2017**

### **Large Displacement Finite Element Analysis**



**Altair**

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## **CONTENTS**

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**13.0 REFERENCES**

**2**

### 3.0 REFERENCES

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